

Physics 264: Problem Set 1

Sean Carroll, Fall 2005

Due Thursday 6 October, 12:00 noon

1. (Hartle 2-5; 25 points) Calculate the area of a circle of radius r (distance from center to circumference) in the two-dimensional geometry which is the surface of a sphere of radius a . Show that this reduces to πr^2 when $r \ll a$.

2. (Hartle 2-7; 50 points) Consider the following coordinate transformation from the familiar rectangular coordinates (x, y) labeling points in the plane to a new set of coordinates (μ, ν)

$$x = \mu\nu, \quad y = \frac{1}{2}(\mu^2 - \nu^2). \quad (0.1)$$

(a) Sketch the curves of constant μ and curves of constant ν in the (x, y) plane.

(b) Transform the line element $ds^2 = dx^2 + dy^2$ into (μ, ν) coordinates.

(c) Do the curves of constant μ and constant ν intersect at right angles? (Provide a justification, not just an answer.)

(d) Find the equation of a circle of radius r centered at the origin in terms of μ and ν .

(e) Calculate the ratio of the circumference to the diameter of a circle using (μ, ν) coordinates. Do you get the correct answer?

3. (Hartle 3-1; 25 points) Show that Newton's laws of motion are *not* invariant under a transformation to a frame that is uniformly accelerated with respect to an inertial frame. What are the equations of motion in the accelerated frame (say, accelerated in the x -direction)?