Cosmological Structure Formation and Large Scale Magnetic Fields

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Since they were first observed in the late 1940s, astrophysical magnetic fields have been a puzzle for physicists. The large strengths and scales of these fields led to many questions concerning their creation and evolution. In this report I discuss a recent paper that addresses the former problem. In Cosmological Structure Formation Creates Large-Scale Magnetic Fields *authors Siegel and Fry show that cosmological perturbations lead to tiny magnetic seed fields. They theorize that these fields may be the origins of the fields observed today. Beyond the examination of this article, a brief summary of existing observations and a brief discussion of the article's relation to other research are included to place this work in the proper context.*

1 Introduction

The first extraterrestrial magnetic field was observed in sunspots by Hale in 1908 [1]. Since then magnetic fields have been observed in galaxies (1949) [2], galaxy clusters and potentially in superclusters [1]. The existence of these strong fields (on the order of a few μG) on such a range of scales is an unresolved issue in astrophysics today. Two important and often debated aspects of this puzzle are the origins of these fields and the physical processes by which they evolved. In this report I discuss Siegel and Fry's *Cosmological Structure Formation Creates Large-Scale Magnetic Fields* [3] (from now on referenced as SF 2006), a paper that attempts to answer the former question.

There are two basic categories into which magnetic field formation theories fall: those that rely on astrophysical processes and those that require exotic new physics. Theories based on astrophysical processes (such as the one proposed in SF 2006) generally have seed magnetic fields arising from processes involving the difference in mobility between electrons and ions [1].

Theories based on exotic physics, in contrast, rely on new physics in the early universe. For example, it is theorized that magnetic fields could arise during the electroweak or QCD phase transitions.

A common difficulty encountered by theories in both categories is their inability to predict strong enough fields on large enough scales [1], [4]. Part of this failure arises from uncertainties in how these early seed fields are amplified into the fields observed today. The standard theory of field amplification uses an $\alpha\Omega$ (*helical turbulence-differential rotation*) dynamo to amplify and regenerate large scale magnetic fields [3], [1]. However, problems have arisen with this theory, so it is uncertain whether such a dynamo could adequately amplify small seed fields to the strength and scales observed today [1]. Many alternative theories and modifications have been proposed (cf. [4], [5], [6] for example)

This report will be organized as follows: first, the observational data that provides evidence for and constraints on early magnetic fields is summarized. Next, the ideas presented in SF 2006 are throughly examined. Finally, the work of Siegel and Fry is briefly contrasted to a related competing hypothesis for the genesis of these early fields.

2 Background

2.1 Observational Evidence

Since the first galactic magnetic fields were observed in 1949, astronomers have searched for evidence of astrophysical magnetic fields with sizes up to the largest scales in the universe[1]. The results of these searches have been mixed. Magnetic fields on order of 10 μG have been observed in spiral galaxies "whenever the pertinent observations are made" [1]. These fields have a coherence length roughly the length of the galaxy. Magnetic fields of roughly the same strength have also been detected in elliptical galaxies, though the coherence length in these galaxies is significantly shorter than the length of the galaxy itself. Firm measurements of the fields in the elliptical galaxies is difficult due to the low density of relativistic electrons (whose interactions with the field create measurable phenomena) [1].

On scales larger than galaxies, Faraday rotation measurements have been used to detect magnetic fields of 0.2 to 3 μG in galaxy clusters. In 1989 Kim *et al.* reported detecting a 0.2- 0.6 μG radio-emission bridge $1.5h_{75}^{-1}$ Mpc in projection between Coma and Abell 1367 [1]. Other observations such as

[1]For a brief summary of how these fields are detected see the appendix

those of radio galaxy NGC 315 in 2001 by Ensslin *et al.* also suggest the presence of such large magnetic fields [1]. However, magnetic fields on such scales are not as ubiquitous as are fields in galaxies. In 1989 Vallée reported that his search for magnetic fields in the Great Attractor Supercluster of galaxies had found no detectable field. This null result placed a limit on of .1 μG on the maximum field strength present [2].

The existence of cosmological fields not associated with collapsed or collapsing structure is still uncertain. Widrow points to the Coma supercluster fields and data from high redshift radio galaxies as hints that these fields might exist. The existence of such large fields supports theories that place the origins of todays observed fields in primordial seed fields amplified by protogalaxy collapse [1]. However, work reported by Vallée in 1991 on magnetic fields in the Bootes Void (diameter 120 Mpc) placed a strict limit of 0.1 μG on the field strength present in that region [2]. Additionally, in 1990 Vallée tested for magnetic fields on the scale of the universe and placed an upper limit of 1 nG for the strength of such a field [2].

2.2 Observational Constraints

Observations of the early universe also provide information about astrophysical magnetic fields. Our knowledge of the Cosmic Microwave Background's (CMB's) angular power spectrum places constraints on early magnetic fields. If a magnetic field greater than the horizon at the time of recombination had existed it would have effected the distribution of structure in the universe. Widrow describes the process in the following way [1]: consider a homogenous universe with a unidirectional magnetic field. Expansion along the direction of the field stretches the field lines and, as a consequence, does work. However, expansion perpendicular to the field lines is facilitated by magnetic pressure. Therefore, the universe expands more slowly along the direction of the magnetic field. If such an expansion occurred than we would observe the redshift of objects along the field lines be less than those of objects of similar age receding perpendicular to the field lines. However, we know from COBE and WMAP data that our universe is very homogenous and isotropic. Using this COBE data Barrow, Ferreira, and Silk determined that any field having a comoving distance greater than 10 Mpc would currently have at most a strength of 10^{-8}G [1].

Big Bang nucleosynthesis data also places constraints on early magnetic fields. Magnetic fields have two effects on nucleosynthesis: nuclear reaction rates change in the presence of strong magnetic fields and, more significantly, the energy density from the magnetic fields leads to an higher cosmological expansion rate than the standard theoretical model [1]. The increased expansion rates leads to an increased ^{4}He abundance. From measurements of the ^{4}He abundance physicists have determined that the strongest cosmological field that could have existed during this time was 6 T G. This field would evolve to a field of 1 μG today [1], a somewhat less stringent restriction than that imposed by the CMB.

3 Cosmological Structure Formation and the Creation of Magnetic Fields

3.1 Summary

In SF 2006, Siegel and Fry seek to show that small seed magnetic fields (of order 10^{-30} G) are necessary consequences of structure formation. To achieve this result, the authors first model the universe as a cosmological fluid. Next, drawing heavily on Ma and Bertschinger's work on perturbation theory [7], they calculate the development of the various fluid components (photons, cold dark matter (CDM), etc), paying special attention to the evolution of photons, electrons, and protons. These evolution equations require that tiny charge separations in the fluid occur and that this asymmetry in charge distribution extend to all scales. Simulations show that this inhomogeneous charge density undergoes small oscillations, creating currents and thus magnetic fields.

3.2 The Evolution of Perturbations

To derive the development of perturbations in the early universe, Ma and Bertschinger consider a spatially flat (Ω =1) homogenous FRW universe with only isentropic scalar metric perturbations. The effect of such perturbations on the metric is easily seen in the conventionally used synchronous gauge:

$$ds^2 = a^2(\tau)\{-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\}$$

where δ_{ij} is the Kronecker delta, h_{ij} is the metric perturbation, a the scale factor, and τ the conformal time given by:

$$\tau = \int \frac{dt}{a}$$

where t is the proper time.

While the conventional synchronous gauge shows the effect of the perturbations most transparently, the conformal Newtonian gauge greatly simplifies calculations (because the metric $g_{\mu\nu}$ is diagonalized):

$$ds^2 = a^2(\tau)\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i\}$$

where ψ and ϕ are metric perturbations. The major disadvantage of the conformal Newtonian gauge is that it is a restricted gauge since the vector and tensor modes are eliminated. However, when dealing with strictly scalar mode perturbations, this restriction is of no consequence.

Having chosen a metric it is straightforward to derive the relevant Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}a^2\bar{\rho}$$

$$\frac{d}{d\tau}\left(\frac{\dot{a}}{a}\right) = -\frac{4\pi}{3}Ga^2(\bar{\rho}+3\bar{P})$$

where $\bar{\rho}$ is the average energy density and $\bar{P}$ is the average pressure, and $\dot{a}$ is the derivative of a with respect to τ.

Taking the Fourier transform of these equations and expanding them to first order in the perturbation yields:

$$k^2\phi + 3\frac{\dot{a}}{a}\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) = 4\pi Ga^2\delta T^0{}_0 \tag{1}$$

$$k^2\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) = 4\pi Ga^2(\bar{\rho}+\bar{P})\theta \tag{2}$$

$$\ddot{\phi} + \frac{\dot{a}}{a}(\dot{\psi}+2\dot{\phi}) + \left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi-\psi) = \frac{4\pi}{3}Ga^2\delta T^i{}_i \tag{3}$$

$$k^2(\phi-\psi) = 12\pi Ga^2(\bar{\rho}+\bar{P})\sigma \tag{4}$$

with θ,(the divergence of the fluid velocity in the case of a perfect fluid) and σ, the shear stress given by:

$$(\bar{\rho}+\bar{P})\theta \equiv ik^j\delta T^0{}_i, \qquad (\bar{\rho}+\bar{P})\sigma \equiv -(\hat{k_i}\hat{k_j} - \frac{1}{3}\delta_{ij})\Sigma^i{}_j$$

where $\Sigma^i{}_j$ is the traceless component of $T^i{}_j$.

The conservation of energy-momentum implied by the Einstein equations places a final restriction on the evolution of perturbations:

$$\nabla_\mu T^{\mu\nu} = 0. \tag{5}$$

For a perfect fluid, the evolution equations are particularly simple. The energy-momentum tensor of a perfect fluid with four-velocity U^μ is given by:

$$T^\mu_{\ \nu} = Pg^\mu_{\ \nu} + (\rho + P)U^\mu U_\nu. \tag{6}$$

Using equations 1 - 6 we can write down the equations of motion governing the behavior of perturbations in a perfect fluid:

$$\dot{\delta} = -(1+\omega)(\theta - 3\dot{\phi}) - 3\frac{\dot{a}}{a}\left(\frac{\delta P}{\delta \rho} - \omega\right)\delta \tag{7}$$

$$\dot{\theta} = -\frac{\dot{a}}{a}(1-3\omega)\theta - \frac{\dot{\omega}}{1+\omega}\theta + \frac{\delta P/\delta\rho}{1+\omega}k^2\delta - k^2\sigma + k^2\psi \tag{8}$$

where $\omega \equiv P/\rho$, $\delta \equiv \delta\rho/\bar{\rho}$ and $\delta P/\delta\rho = c_s$, the adiabatic speed of sound in the fluid.

The stipulation of a perfect fluid is not as great a restriction as it may first seem. CDM, for example, is well approximated by such a fluid. Additionally, more complicated species, such as baryons, can be approximated using the perfect fluid equations with additional terms added to account for coupling to other substances.

While the perfect fluid approximation is sufficient for baryons and CDM, more care must be taken calculating the equations of motion for photons and neutrinos. To determine the evolution of these particles we must integrate the Boltzmann equation. For brevity the derivation of these equations will be omitted here; interested readers are referred to [7] sections 5.1 -5.5.

The baryon evolution equations for protons and electrons lead to the charge separations responsible for the genesis of magnetic fields. As previously stated, these equations are well approximated by those of a perfect fluid with additional terms added to account for Coulombic and photon interactions. In Ma and Bertschinger's approximations the photons interact with electrons (proton interactions are so greatly suppressed compared to electron interactions as to be insignificant) through Thomson scattering. This approximation implies a photon energy $\ll m_e$, the electron's rest energy, and therefore is valid for periods later than neutrino decoupling. Siegel and Fry take the derivations from [7] one step further by considering the proton and electron evolution equations separately. The behavior of these particles is governed by the Poisson equation:

$$\nabla^2\Phi = -\nabla \cdot \vec{E} = 4\pi\rho_c$$

(where ρ_c is the electric charge density), and by Euler's equation of motion for a perfect fluid, modified by a collision operator, C to account for interactions:

$$\frac{1}{a}\frac{\partial a\vec{v}}{\partial t} + \frac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\frac{1}{a}\nabla\phi - \frac{q}{m}\frac{1}{a}\nabla\Phi + C$$

where $\vec{v}$ is the velocity. Using these results the evolution equations of electrons and protons are then:

$$\dot{\delta_e} = -\theta_e + 3\dot{\phi} \tag{9}$$

$$\dot{\theta_e} = -\frac{\dot{a}}{a}\theta_e + c_s^2 k^2 \delta_e + k^2 + \Gamma_e(\theta_\gamma - \theta_e) - \frac{4\pi e^2}{m_e}(n_p - n_e) \tag{10}$$

$$\dot{\delta_p} = -\theta_p + 3\dot{\phi} \tag{11}$$

$$\dot{\theta_p} = -\frac{\dot{a}}{a}\theta_p + c_s^2 k^2 \delta_p + k^2\psi + \Gamma_p(\theta_\gamma - \theta_p) + \frac{4\pi e^2}{m_e}(n_p - n_e) \tag{12}$$

$$n_i = \textit{the number density of species i.}$$

with the photons damping the evolution of electron perturbations by a factor

$$\Gamma_e \equiv \frac{4\bar{\rho}_\gamma n_e \sigma_T a}{3\bar{\rho}_e}$$

where σ_T is the Thomson cross-section. Proton evolution fluctuations are similarly damped, but damping is a factor of 10^{10} smaller, due to the smaller scattering cross-section with photons.

The authors then define several variables to simplify the equations into a form where the charge separation becomes evident.

$$\delta_q = \delta_p - \delta_e, \qquad \theta_q = \theta_p - \theta_e$$

$$\theta_b \equiv \frac{m_e}{m_b}\theta_e + \frac{m_p}{m_b}\theta_p$$

where $m_b = m_p + m_e$. Using these new variables and the facts that $\Gamma_p \ll \Gamma_e$ and $m_b \approx m_p$ the authors then combine equations 9 - 12 to find [2] :

$$\dot{\delta_q} = -\theta_q \tag{13}$$

[2] Unfortunately, I find this important step of the derivation nontransparent. In particular, the authors fail to state the simplifying assumptions they use to generate the term $\frac{4\pi}{m_e} n_e e^2 \delta_q$, which they state arises from Coulombic interactions between the particles. This lack of clarity is particularly bothersome because this term plays an important role in the later simulations of field formation.

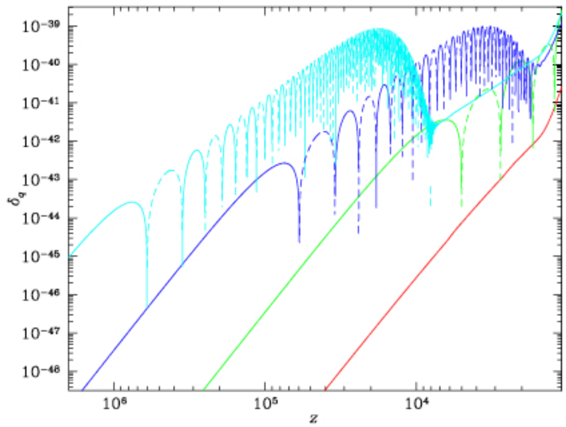

FIG. 1.— Fourier mode amplitudes of the charge asymmetry δ_q plotted as a function of redshift z for comoving scales of (from top to bottom) k = 10, 1, 0.1, and 0.01 Mpc^{-1}. The amplitude δ_q rises as $\sim a^5$ initially, then ceases to grow when the scale of interest enters the horizon, and oscillates at an amplitude which first continues to rise slowly, then falls, eventually matching on to the equilibrium solution $\delta_q \propto \theta_b$. Solid lines indicate positive values of δ_q, and dashed lines indicate negative δ_q.

Figure 1: reproduced from Siegel and Fry, 2006.

$$\dot{\theta_q} = -\frac{\dot{a}}{a}\theta_q + c_s^2 k^2 \delta_q + \frac{4\pi}{m_e} n_e e^2 \delta_q - \Gamma_e(\theta_\gamma - \theta_b + \theta_q) \qquad (14)$$

The term $-\Gamma_e(\theta_\gamma - \theta_b + \theta_q)$ as the authors emphasize *"will create a local charge asymmetry, even when there is none initially."* Choosing a ΛCDM universe with parameters $H_o = 71kms^{-1}$, $\Omega_m = .27$, $\Omega_b = .044$ and a Helium mass fraction Y = .248, Siegel and Fry use COSMICS, a simulation program developed by Bertschinger, to numerically evaluate the evolution of the charge asymmetry (See Figure 1 reproduced from their paper). The illustration of charge asymmetries on a range of scales is exciting to note, for it implies that fields produced by these asymmetries should also occur on all scales. The simulations were run up to the epoch of recombination. Around this epoch photon damping of perturbations becomes increasingly important, however recent work has shown that Silk damping only affects scales with k> 1Mpc^{-1}, so the large scale charge asymmetries remain.

3.3 Magnetic Field Formation

Simulations of the evolution of charge asymmetries provides data on both local currents and bulk movements of charge. Recalling from Maxwell's equations the relation between currents and magnetic fields:

$$\nabla \times \vec{B} = 4\pi\vec{J} + \frac{\partial\vec{E}}{\partial t}$$

where the current density, $\vec{J}$ is given by

$$\vec{J} = n_p e\vec{v_p} - n_e e\vec{v_e} \simeq n_e e(\delta_q\vec{v_b} + (1+\delta_b)\vec{v_q})$$

and $\vec{v_q}$ is the difference between the proton and electron velocity, we see that the evolution of charge asymmetries *necessitates* the existence of magnetic fields. Moving into the Fourier domain the power spectrum of the field can be calculated. The peak of the spectrum corresponds a field strength of 10^{-30}G on a comoving scale of $0.1Mpc^{-1}$. Because of the early time of the field's origin, this size and scale is just large enough by some estimates to evolve into today's large scale fields [8].

4 Conclusion

This result obtained by Siegel and Fry appears to provide a good candidate for the origins of the universe's large scale magnetic fields. However, this work, like many other early universe theories, has its competitors. One particularly interesting alternative theory was recently discussed in Science. In this report, Ichiki *et al.* also look to density perturbations as a source of early magnetic fields [9]. Like SF 2006, they claim that around decoupling the interactions between photons and baryons are weak enough that small electric currents (and thus magnetic fields) are created by the difference in mobility of protons and electrons. In addition, drawing on previous research, they require vorticity of plasma to produce the current. As such vorticity is not produced in first order perturbations [10], higher order perturbation couplings must be considered.

Using a model which has three main contributors to the generation of magnetic fields: (i) scattering between photons and charged particles, (ii) the vorticity difference term and (iii) the anisotropic pressure (from photons), Ickiki *et al.* numerically estimate that these process could lead to early fields as strong as 10^{-24} G at scales of 1 Mpc [9]. This process seems markedly more successful at producing strong large scale fields than that of

SF 2006. Siegel and Fry counter by pointing out that all the field magnitudes are determined by velocities and that higher modes of perturbation theory should have less velocity than the first order gravitational instabilities [3]. Thus, they claim that the fields produced by the mechanisms of Ickiki *et al.* should in fact be *weaker* than the fields obtained by using first order perturbations.

Ultimately, the validation of one theory over another will be achieved through observations. Both of these theories have observable consequences. The SF 2006 power spectrum predictions for large scale fields at recombination should be observable. Unfortunately, the best current hope for observating this spectrum, PLANK, has a predicted lowest field sensitivity of $\approx 10^{-10}$ G, not nearly good enough to see the 10^{-30}G fields predicted [3]. For his part, Ichiki suggests that gamma ray burst observations would allow detection of the fields they predict. (The presence of magnetic fields can alter the arrival of gamma ray photons because the high energy electrons responsible for such fields are deflected by the fields.) While I find this proposal somewhat suspect, due to the weak nature of the fields involved, Ichiki believes that future gamma ray experiments such as GLAST could conceivably measure the weak early universe fields predicted.

In conclusion, the work of Siegel and Fry and others illustrates how density perturbations are well suited to be the creators of cosmological magnetic fields. The range of length scales involved, the long time frame allowed for the fields to be amplified (from recombination to today) and the apparent success of these theories at predicting fields of adequate scales and lengths all make these perturbations a promising solution to a long unsolved problem in astrophysics.

5 Appendix: Detecting Magnetic Fields

There are four main physical phenomena that allow us to detect astrophysical magnetic fields: the Zeeman effect, polarized starlight, synchrotron radiation and Faraday rotation[3]. The limitations inherent in using each of these processes puts important limits on measurements of the presence and strengths of magnetic fields.

Zeeman splitting, the effect used by Hale to detect magnetic fields on the sun, is a quantum mechanical effect that occurs when a magnetic field breaks atomic energy level degeneracies. The splitting of the energy levels is given by:

$$\Delta E = \textit{(Landè g factor)(Bohr magneton)}B$$

Because the energy level splitting is convoluted with thermal level broadening, the Zeeman effect can only be seen in regions of low temperature and high magnetic field[1].

Polarized starlight is created by polarized absorption of light by dust grains aligned in a magnetic field[1]. Polarized starlight allows us to detect magnetic fields in the Milky Way and neighboring galaxies, but because this effect depends on extinction it is not suitable for detecting distant magnetic fields[1].

Synchrotron emission is radiation created by relativistic electrons moving in a magnetic field[1]. This radiation allows us to observe magnetic fields on scales up to superclusters. However, its usefulness in determining field properties is limited by our theoretical understanding of how total synchrotron emission and polarization relate to magnetic field properties when other astrophysical phenomena that effect electrons, (such as cosmic rays), are present.

Finally, Faraday rotation provides information on the magnetic field in the foreground of an object. Light waves undergo Faraday rotation when passing through space containing a magnetic field and free electrons[1] . In this region left and right circularly polarized light travel with different phase velocities and this causes the polarization of the wave to rotate. This rotation angle, β and the more commonly reported rotation measure, RM, are given by[11]:

$$\beta = RM\lambda^2$$

with

$$RM = \frac{e^3}{2\pi m^2 c^4} \int_0^d n_e B ds$$

where

$$e = \textit{electron charge},\ m = \textit{electron mass},\ c = \textit{speed of light in vacuum},$$
$$n_e = \textit{electron number density},\ d = \textit{propagation distance}$$

It takes three or more wavelength measurements to remove any $n\pi$ degeneracies in β and to determine RM[1]. Light from distant sources may be rotated several times (by the source, the IGM, etc.) before reaching detectors on Earth. For example, incoming 435 MHz radio waves experience 1.5 rotations from Earth's ionosphere alone [11].

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