Cosmology with Candles and Sirens

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Abstract

Measurements of type Ia supernova "standard candles" have led us to the striking conclusion that the universe is currently accelerating in its expansion, presumably driven by some unknown energy source (dubbed "dark energy"). However, given the absence of a solid theoretical model for Type Ia supernovae, it is difficult to rule out systematic effects such as evolution of supernova brightness that could undermine the validity of these candles (and the conclusion that the universe is accelerating). Therefore, it is of great interest to have additional, independent standard candles to confirm (or deny) our current picture of the universe. Gravitational-wave-driven binary black hole (or neutron star) inspirals, as measured by LIGO or LISA, are a promising standard candle, if a redshift-providing electromagnetic counterpart to the inspiral can be identified. In this paper, I review supernovae techniques and survey current work on the feasibility of using such gravitational wave "standard sirens".

1 Introduction and Outline

A standard candle is an object whose intrinsic brightness is known, so that by measuring the observed brightness one can infer the (luminosity) distance to the source. If one can additionally determine the redshift of the object, such candles can be used to map out the distance-redshift relation and thereby constrain cosmological parameters. Type Ia supernovae have proved effective standard candles, and led to the remarkable discovery that the universe is accelerating in its expansion. However, given the lingering possibility of systematic effects, it is useful to have additional standard candles that could independently verify the distance-redshift relation. Recent work has looked in to the possibility that binary compact object inspirals (a compact object is a black hole or neutron star) could be used as standard candles (and in such a case they would be named 'standard sirens'). The primary obstacle to doing cosmology with such systems is that while their gravitational waves encode luminosity distance, there is no redshift information present in the signal. In order to learn the redshift of such a system there would have to exist an electromagnetic counterpart to the inspiral that could be identified and spectrally analyzed. At present it is unknown if such a counterpart exists, but there is reason to suspect that it might for certain types of inspirals.

In section 2, I review the use of Type Ia supernovae as standard candles and discuss sources of systematic error. In section 3, I cover the basics of binary inspiral theory and address distance determination from gravitational waves. In section 4 I discuss the potential for standard sirenhood (and resulting cosmological uses) of each of the three types of binary inspirals we expect to measure. Finally, in section 5 I summarize and conclude.

2 Supernova Review

Historically, supernovae have been classified by their spectra, and the name "type Ia" refers to supernovae with no hydrogen, and with a silicon abundance. It is currently believed that these supernovae are caused by the thermonuclear explosion of an accreting white dwarf star. When the mass of the star exceeds the Chandrasekhar limit of 1.4 $M_\odot$, the star's self-gravity becomes larger than the electron degeneracy pressure supporting it, and matter begins to collapse, igniting nuclear fusion. The details of ignition are not well understood, and simulations suggest a variety of ways in which the explosion could commence and proceed, with no scenario winning over any other. However, predictive simulations (or any sort of theoretical understanding at all) are not necessary to use an object as a standard candle, if the intrinsic luminosity of the object can simply be determined from observations.

Luminosity drops off as the square of the distance, and therefore one can determine the intrinsic luminosity of an object at known distance by simply measuring the observed luminosity and correcting appropriately. In the early 1990's, a project was undertaken to measure type Ia supernovae at known distances, in order to determine if they were indeed standard (i.e., if the intrinsic brightness of a type Ia supernovae is indeed always the same). After the observation of thirty nearby supernovae in 1994, it unfortunately became clear that though type Ia's were a homogeneous bunch when compared to other kinds of supernovae, there was still too much variation in intrinsic brightness for them to be uses as standard candles. However, it was subsequently discovered that the *shape* of the light curve (luminosity as a function of time) of such supernovae was in fact correlated to their intrinsic brightness! The reason for this correlation is still unknown, but it can nevertheless be used to systematically "standardize" observed type Ia supernovae, by which is meant simply that one can infer the intrinsic brightness of an individual supernovae from the shape of its light curve. Then intrinsic brightness can be compared to measured brightness in the usual way to yield a luminosity distance. Figure 1 demonstrates the striking efficacy of this standardization procedure.

The obvious assumption underlying the above analysis is that distant supernovae have the same properties (in particular this miraculous correlation) as the nearby ones at known distance. This assumption is ultimately not justifiable given the total lack of a predictive theory of type Ia supernovae, but there is good reason to believe that

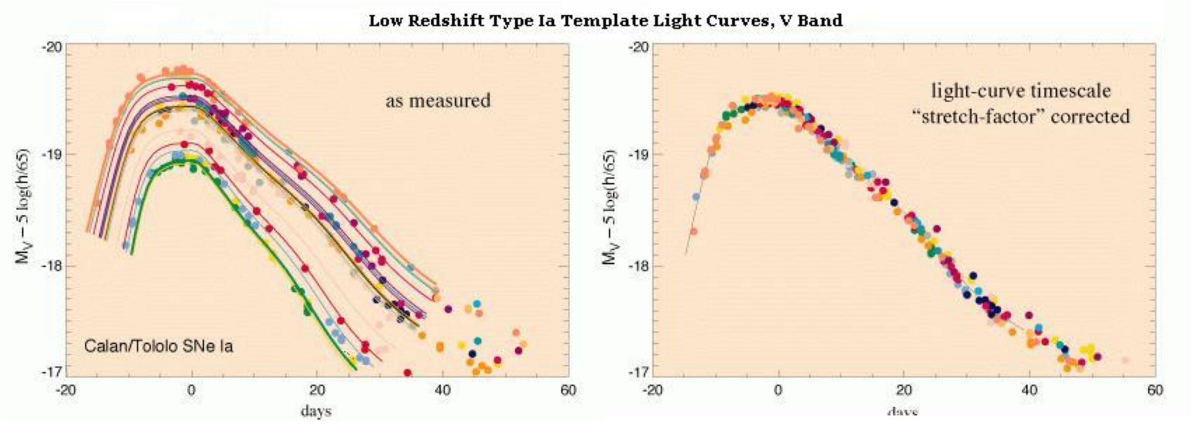

Figure 1: Left: light curves of nearby supernovae, and right: "standardization" of these curves by the stretching method of the Perlmutter group. A simple stretching function of the light curve shape mimics a standard curve astonishingly well in the V band, indicating that shape and peak brightness are tightly correlated.

it holds. The main supporting observation is that while nearby supernovae do show differences correlating to the age of the host galaxy (i.e., whether there is star formation or not), this correlation disappears upon standardization. While the intrinsic brightness of a supernova seems to be sensitive to the age its host galaxy, our ability to determine that brightness is not. Therefore, we should not be worried that our standardization is thrown off by observing a greater fraction of younger host galaxies at higher redshift.

There are three major sources of error in supernovae measurements: dust extinction, the Malmquist bias, and gravitational lensing. Dust extinction refers to the absorption and scattering of photons by dust particles in the host galaxy and in our Milky Way. All wavelengths of light are absorbed (dimmed) in roughly equal proportion, but short wavelengths scatter (out of our line of sight) more frequently than long wavelengths, creating a tell-tale reddening of the signal. If light from supernovae is measured at a large enough span of wavelengths, this reddening can be used to infer the amount of dimming from extinction and correct for it (but still introducing some error in to the calculation). The Malmquist bias refers to the selection effect whereby experiments tend to detect brighter than average objects at distances that correspond to near their (lower) sensitivity limit. The errors from this effect can be Monte-Carlo estimated and accounted for. Finally, gravitational lensing refers to the (de-)magnification of light that encounters an inhomogeneous mass distribution on its way to Earth. It is unlikely that we will know the mass distribution with enough certainty to manually correct for lensing (in fact, lensing from known sources is typically used to *probe* the mass distribution), and therefore this introduces an error that rapidly increases with redshift. It is likely that lensing effects will ultimately limit the efficacy of supernovae cosmology.

3 Binary Inspiral Theory

Although all accelerating masses should emit gravitational waves, only neutron stars and black holes have sufficient mass for the resulting radiation reaction force to significantly modify their orbits. Binary systems of such compact objects should spiral inwards under gravitational radiation reaction and eventually merge to form a single compact object. In analyzing this process, one divides in to three stages: inspiral, merger, and ringdown. In the inspiral stage, the objects are separated by many object radaii and evolve adiabatically under a gentle gravitational self-force. When the orbital separation decreases to the order of an object radius, we enter the violent merger stage: evolution becomes non-adiabatic and coalescence ensues. The moments before assumption of the final stable state (a single object) are known as "ringdown" because the waves take the form of damped sinusoids, much like those of a bell. Although we expect to detect systems in all stages of evolution, the inspiral phase provides the best distance information and is thus most important for cosmology.

Inspiral is well-modeled by the parameterized post-Newtonian approximation to general relativity. The analysis reveals that the dominating harmonic of incoming waves is the $l = 2, m = 2$ harmonic, and the two independent polarizations take the form

$$h_+ = \frac{2\mathcal{M}^{5/3}\left[\pi f\left(t\right)\right]^{2/3}}{D_L}\left[1+\left(\hat{L}\cdot\hat{n}\right)^2\right]\cos\left[\Phi\left(t\right)\right], \tag{1}$$

$$h_\times = \frac{4\mathcal{M}^{5/3}\left[\pi f\left(t\right)\right]^{2/3}\left(\hat{L}\cdot\hat{n}\right)}{D_L}\sin\left[\Phi\left(t\right)\right]. \tag{2}$$

$\Phi(t)$ is the gravitational-wave phase function as measured by the interferometer, and $f(t) = 2\pi(df/dt)$ is likewise the measured frequency. $\hat{L}$ is the direction of the binary's angular momentum, and $\hat{n}$ is its position on the sky; these angles can be determined if the wave is measured from more than one location (using multiple earth-based detectors, or one orbiting space-based detector). The quantity $\mathcal{M}$ is the binary's (redshifted) "chirp mass",

$$\mathcal{M} = \left(1+z\right)\frac{\left(m_1 m_2\right)^{3/5}}{\left(m_1+m_2\right)^{1/5}}. \tag{3}$$

which in fact depends strongly on the measured phase through an equation of the form

$$\Phi\left(t\right) = \alpha + \beta\left(\mathcal{M}f\right)^{-5/3}, \tag{4}$$

where α and β are known constants. Therefore, the remaining quantity in the equations, the luminosity distance D_L, can be calculated from measurements of just this harmonic (and using fainter harmonics increases the accuracy of the calculation).

The fact that the chirp mass is a redshifted quantity is indicative of a general feature: because general relativity has no absolute scale, information about intrinsic parameters of the binary (i.e. its masses and spins) is always redshifted. For example, in a system where gravity is the only force impacting evolution, a mass parameter m can only influence the evolution as the time scale $\tau_m = Gm/c^3$. If the system is cosmologically distant, the time scale in the signal we measure is redshifted, and mass parameters therefore “pick up” a factor of $1+z$. The same argument applies for spins S, which enter as a squared timescale ${\tau_s}^2 = GS/c^4$, and thus pick up a factor of $(1+z)^2$. We cannot in principle disentangle redshift from masses and spins; it is impossible to gain redshift information (or mass/spin information, for that matter) from gravitational wave measurements alone.

4 Possible Standard Sirens

Indirect detection methods suggest that there are essentially two classes of compact objects in our universe: little ones and big ones. Little ones are neutron stars and black holes that have formed from collapsed stars; big ones are supermassive black holes at the centers of galaxies whose origin can only be speculated at. Mixing and matching these types, there are therefore three classes of binary compact object systems: little-little, big-little, and big-big. Little-little systems will be measured by ground-based detectors, whereas big-little and big-big will be measured by LISA. Following, I address the cosmological potential of each of these classes of system, in terms of the redshift of the system, the likelihood of identifying an electromagnetic counterpart, the expected event rate, and the precision to which luminosity distance can be determined.

4.1 Little-little systems

Little-little systems have the advantage that we already know they exist, for we have detected them in our galaxy (radio measurements of one such system won Hulse and Taylor the 1993 Nobel Prize for indirect detection of gravitational waves). Of the various combinations of stellar compact objects, a double neutron star binary stands out for investigation because of a probable electromagnetic couterpart: it is currently believed that neutron star mergers are responsible for no less than short gamma-ray bursts! If this is indeed true, than we will measure many such electromagnetically complimented standard sirens in the first years of advanced LIGO, spelling good news for cosmology.

The drawback is that these sources will be at low redshift; ground-based detectors will measure gravitational waves from neutron star binaries at distances of no more than a few hundred MegaParsec. However, as emphasized by Hu [5], even such low redshift ($z \lesssim 0.2$) sources constrain dark energy when considered in light of CMB measurements. Assuming a flat universe as indicated by the CMB, the angular diameter distance to a redshift z is given by

$$D_A(z) = (1+z)^{-1} \int_0^z \frac{dz'}{H(z')}, \tag{5}$$

where $H(z)$ is expressed in terms of the density parameters as

$$H(z) = H_0 \sqrt{\Omega_m (1+z)^3 + \Omega_{de} (1+z)^{3(1+w)}}. \tag{6}$$

Here we have assumed negligible contribution from radiation and a constant dark energy equation of state parameter w. However, given that matter and dark energy are then the only components of this universe, we additionally have that $\Omega_{de} = 1 - \Omega_m$, leaving five unknown parameters: D_A, z, h, Ω_m, and w. The heights of the acoustic peaks in the CMB fix $\Omega_m h^2$, and the locations the peaks (i.e., their angular scale) determine the angular diameter distance D_A (to the last scattering surface at $z = 1100$). If h can be determined accurately by low-redshift measurements, then the only the remaining unknown, w, is given by the pair of equations. It is in this way that low-redshift candles, such as the putative neutron star mergers accompanied by short gamma-ray bursts, can constrain dark energy. Dalal, Holz, Hughes, and Jain [3] consider the feasibility of neutron star standard sirens in terms of today's detectors, and conclude that once ground-based detectors LIGO, Virgo, and AIGO reach expected LIGO-II sensitivity, h can be measured at the 2% level after a single year of observation (assuming that short gamma-ray bursts are indeed caused by neutron star mergers).

4.2 Big-little systems

The proper name for a system consisting of a big black hole and a little black hole (or other compact object) is an “Extreme Mass Ratio Inspiral”, or EMRI. The inspiral corresponds to the situation where a compact object formed in a galaxy passes too close to that galaxy’s central central black hole and is swallowed up. EMRI’s should be less common than little-little mergers, but the increased power in their waves (see equation 3, setting the amplitude) will increase the distance to which we are sensitive to them, resulting in a detection rate of at least several to possibly hundreds per year. The space-based detector LISA is design-optimized for detection of these events, and most events will come from redshifts of approximately .3 to .8, which is perfect for directly constraining dark energy. However, it is unlikely that these events will ever be used as sirens, for it will likely prove insurmountably difficult to establish their redshift. The existence of an electromagnetic counterpart would be a surprising discovery indeed, for a stellar black hole or neutron star would simply be swallowed up by the supermassive black hole before anything interesting could happen. If the object in orbit were instead a white dwarf, it is possible that tidal forces could rip it apart before it is swallowed, in which case an electromagnetic couterpart is conceivable (but such a counterpart has not yet been conceived).

4.3 Big-big systems

We are far more likely to find an electromagnetic counterpart in the case of a binary of two supermassive black holes. These binaries would be at high redshift, occurring during the era of structure formation when galaxies are frequently colliding and merging. The event rate is poorly understood, but the hope is for at least a few detections over the lifetime of the LISA mission. Although there is no definitive prediction for an electromagnetic counterpart, it is easy to imagine one’s existence during the chaotic and powerful event of a galaxy collision. Two basic scenarios have been proposed. Milosavljevic and Phinney [6] find that the process of inspiral delays accretion of circumbinary gas until a few years after the merger is complete, leading to an x-ray afterglow potentially detectable by the next generation of telescopes. The competing scenario, investigated most recently by Armitage and Natarajan [7], holds alternatively that the inspiral of the smaller member of the binary drives gas on to the larger member, causing super-Eddington accretion and leading to bright optical emission.

Even if a distinct electromagnetic counterpart is not identifiable, it may still be possible to determine the host galaxy of the event and learn the redshift from measurements of that galaxy. LISA cannot possibly localize a source with enough accuracy to directly pinpoint its host galaxy, but its error box for supermassive binary mergers is reasonably small, on the order of tens of arcminutes. Holz and Hughes [4] employ a simple counting argument based on the galaxy density observed in the Hubble Deep Field to discover that one arcminute of LISA error cube corresponds to approximately 10 to 20 galaxies. Thus for good LISA detections there is likely to be a tractable number of potential host galaxies that could be searched not only for electromagnetic counterparts but also for irregular morphologies or other oddities that might follow a galactic merger. If only one such irregular object is found in the error cube (a reasonable expectation with only at most tens of galaxies present), it can be reasonably assumed that this object is the host. For comparison, note that this procedure cannot work in the case of EMRI inspirals, for two reasons: the error box size is much larger, on the order of a degree (!), and there is no reason to expect an irregularity in the host object in the first place.

Holz and Hughes [4] also investigate the precision with which LISA can determine luminosity distances. They find that the improvement in source localization from the identification of an electromagnetic counterpart (or determination of host galaxy by other means) reduces the errors in the luminosity distance determination to the percent level. (See equations 1; the electromagnetic sky localization corresponds to an artificially good determination of $\hat{n}$.) However, gravitational lensing ultimately degrades this precision, as it degrades the precision of all high-redshift standard candles. For the redshifts expected here ($z \lesssim 10$), lensing imparts a depressing 5-10% error on each individual event. Whereas supernova surveys can average lensing away over a series of detections, the low event rate for supermassive binary black hole mergers means that lensing will be an insurmountable source of error. The high redshift of these candles also means that even in the event of many detections little will be done to constrain dark energy, for the redshifts correspond to times well in to the matter-dominated era. Therefore, despite fantastic intrinsic precision, big-big standard candles are more likely to play the role of independent verifiers of supernovae techniques rather than bold hailers of exciting new physics. However, this is an honorable role to play, given the weakness of underlying theories of supernovae.

5 Conclusions

In this paper, I have considered four physical systems for use as standard candles. Type Ia supernovae are powerful probes of dark energy whose only real drawback is the suspicious and unexplained miracle of light-curve brightness correlation. Neutron star binaries accompanied by short gamma-ray bursts allow a precise measurement of the Hubble constant, useful in determining the dark energy equation of state parameter given CMB priors. Extreme mass-ratio inspirals are likely of no use as standard sirens, despite a high detection rate and prefect redshift situation, because redshift information will be difficult if not impossible to glean. Supermassive black hole mergers will potentially have electromagnetic counterparts, but are limited in utility as candles due to a low event rate and high lensing errors. All types of non-supernova standard candles, however, have an important role to play in increasing our confidence in supernovae techniques, whose theoretical foundations (or lack thereof) have yet to be explained.

The future remains bright for the observation of dark energy.

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