Physics 371 Paper
The Effect of Inhomogeneities on the Luminosity Distance–Redshift Relation: is Dark Energy Necessary in a Perturbed Universe?

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June 2, 2006

Abstract

Dark energy is required in order to explain the results of observations of type Ia supernovae in a homogeneous and isotropic universe. If, however, the universe is somewhat inhomogeneous, dark energy may not be necessary. Barausse et al. calculated the d_L–z relation for such an inhomogeneous universe and determined that to account for super-Hubble inflationary perturbations, there are multiple values of d_L which are possible for a given z. They found that the supernova data may be consistent with an overall flat matter-dominated universe if such perturbations are taken into account. Other authors, however, have shown that that paper is probably incorrect. The idea of replacing dark energy with inhomogeneities is still being investigated though.

1 Introduction

One of the most important recent discoveries in cosmology has been that we seem to live in a vacuum dominated ($\Omega_{\Lambda 0} \approx 0.7$, $\Omega_{M0} \approx 0.3$) Friedmann-Robertson-Walker (FRW) universe. This was the result of the study of type Ia supernovae [1], which was based on the standard luminosity distance–redshift (d_L–z) relation.

$\Omega_{\Lambda 0} \approx 0.7$ can be caused by either a *dark energy* fluid, which is almost constant as the scale factor a changes, or a cosmological constant, Λ, in the Einstein equation. It turns out that the energy density, $\rho_{\Lambda 0} = \rho_{c0}\Omega_{\Lambda 0}$, is very small, while the only natural values we would expect are zero or a very large number. Observing a small dark energy density is therefore somewhat undesirable. It would be nice if we simply misinterpreted the supernova results, and actually have a matter-dominated (Einstein-deSitter) universe.

What the paper I have selected [2] suggests is that the d_L–z relation must be modified if we have a perturbed universe rather than a perfectly homogeneous one. The paper then goes on to show that if we consider a range of perturbations caused by inflation of size larger than our Hubble radius (super-Hubble), it is possible that due to cosmic variance, the supernova data may still be consistent with an Einstein-deSitter universe.

However, [2] has met with considerable criticism, and seems to have been disproved. The idea of eliminating dark energy by considering inhomogeneities has remained popular though. More recent papers have investigated sub-Hubble perturbations and underdense regions centered near us. These models, in order to be considered as viable alternatives to dark energy, must also predict any other observed consequences of dark energy, such as the peaks in the CMB power spectrum.

2 Background

Here is a brief summary of FRW cosmolgy, and evidence for dark energy, as discussed in class. For more details, see [3], [4] and [5].

2.1 FRW cosmology

Assuming spacetime is homogeneous and isotropic we obtain the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad , \tag{1}$$

where $a(t)$ is the scale factor, which denotes the relative size of the universe, and the constant k refers to the shape of the universe (flat, hyperbolic, or spherical). This assumption of homogeneity and isotropy is thought to be valid on large enough scales (> 10 Mpc).

The contents of the universe are then assumed to be a perfect fluid with energy-momentum tensor ${T_\mu}^\nu = \mathrm{diag}(-\rho, p, p, p)$ in the rest frame of the fluid, with ρ the energy density, and p the pressure. With an equation of state $p = w\rho$, conservation of $T_{\mu\nu}$ then implies

$$\rho = \rho_0 a^{-3(1+w)} \quad , \tag{2}$$

where we have set $a_0 = 1$.

Einstein's equation is then equivalent to the Friedmann equations,

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \quad , \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \quad , \tag{4}$$

where the Hubble parameter H is defined to be $\dot{a}/a$. The deceleration parameter q is defined as

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} \quad . \tag{5}$$

If $q < 0$ then $\ddot{a} > 0$ so the universe is accelerating.

We usually consider radiation (R), matter (M), and vacuum energy (Λ) as types of fluids in this model. Their equation of state parameters are summarized in Table 1. We can also count curvature (K) in the same manner, pretending it has an energy density $\rho_K = -3k/(8\pi G a^2)$.

Fluid Type	Abbreviation	w
Vacuum	Λ	-1
Curvature	K	$-1/3$
Matter	M	0
Radiation	R	$1/3$

Table 1: Equation of state parameters for different fluid types

Writing the Friedmann equation in terms of the various energy densities, we have

$$H^2 = \frac{8\pi G}{3}\left[\rho_{M0}a^{-3} + \rho_{R0}a^{-4} + \rho_{\Lambda 0}\right] - ka^{-2} \quad . \tag{6}$$

This may be re-written by defining a critical density $\rho_c = 3H^2/(8\pi G)$, as well as density parameters $\Omega_i = \rho_i/\rho_c$. The Friedmann equation then becomes, after dividing through by H^2

$$1 = \Omega_M + \Omega_R + \Omega_\Lambda - \frac{k}{a^2H^2} \quad . \tag{7}$$

Now it is clear that the universe is flat if $\Omega_{TOT} \equiv \Omega_M + \Omega_R + \Omega_\Lambda = 1$.

Finally, the deceleration parameter may be written in terms of the density parameters,

$$q = \frac{\Omega_M}{2} + \Omega_R - \Omega_\Lambda \quad . \tag{8}$$

In particular, for a universe where $\Omega_R = \Omega_\Lambda = 0$,

$$q = \frac{1}{2} + \frac{k}{2H^2a^2} \quad . \tag{9}$$

2.2 Luminosity distance–redshift relation

The next thing to do is try to relate these cosmological parameters to physical observables. Supposing we have a source of luminosity L, we can measure the flux F of radiation here which was emitted from the source. Then we can define the luminosity distance d_L to be the effective distance from here to the source if the universe were static,

$$d_L = \left(\frac{L}{4\pi F}\right)^{1/2} \quad . \tag{10}$$

In reality, the universe is expanding, so d_L is different from the instantaneous distance to the source at any particular time.

We can also measure the redshift z of an object by comparing what we measure to be the emission spectrum of that object with what we would expect it to be. The redshift is defined as

$$z = \frac{\lambda_{observed} - \lambda_{emitted}}{\lambda_{emitted}} \quad . \tag{11}$$

The redshift is caused by the universe expanding and stretching out the electromagnetic waves, so

$$a = \frac{1}{1+z} \quad . \tag{12}$$

The redshift and luminosity distance can easily be related. Suppose some photons are emitted at redshift z. The number of photons remains fixed, but they will be redshifted by z when we observe them, so the energy per photon observed by us will be reduced by a factor of $(1+z)$ due to the stretching of the wavelength. Also, they will arrive less frequently since the time interval between photons is increased by a factor $(1+z)$. So F will be reduced by a factor $(1+z)^2$, and since $d_L \propto F^{-1/2}$, d_L will simply be $(1+z)$ times the comoving distance from the source to the observer. Since we are dealing with light, by considering null paths, $ds^2 = 0$, and using equations (6) and (12), we obtain the result

$$d_L = (1+z)\frac{H_0^{-1}}{\sqrt{|\Omega_{K0}|}} S_k\left[\sqrt{|\Omega_{K0}|}\int_0^z \frac{\mathrm{d}z'}{E(z')}\right] \quad , \tag{13}$$

where

$$E(z) = [\Omega_{M0}(1+z)^3 + \Omega_{R0}(1+z)^4 + \Omega_{\Lambda 0} + \Omega_{K0}(1+z)^2]^{1/2} \tag{14}$$

and

$$S_k(x) = \begin{cases} \sinh x & k=-1 \\ x & k=0 \\ \sin x & k=1 \end{cases} \tag{15}$$

By measuring z and d_L for many sources, we may then determine the constants Ω_{i0} to get the best fit.

Another useful expression for d_L is Hubble's law, which is an expansion of (13) about $z=0$,

$$H_0 d_L = z + \frac{1}{2}(1-q_0)z^2 + O(z^3) \quad . \tag{16}$$

2.3 Type Ia supernovae

To determine the density parameters through (13), d_L and z must be accurately known for many sources. To do this, Riess et al looked at looked at type Ia supernovae at redshifts in the range $0.16 \leq z \leq 0.97$ [1]. Type Ia supernovae occur when a white dwarf which is accreting matter from a nearby star reaches its Chandrasekhar limit, which is known to be roughly 1.44 solar masses. As such, they serve as good standard candles. Since the luminosity is well known, d_L can be determined by measuring the flux.

Since $\rho_R \propto a^{-4}$, radiation energy density is very low today, so it can be ignored. Galaxy rotation curves show that $\Omega_{M0} \approx 0.2$, so (13) was tested with $(\Omega_{M0}, \Omega_{\Lambda 0}) \approx (0.2, 0.0)$. The results were that the measured luminosity distances were higher than predicted. Next, leaving Ω_{M0} and $\Omega_{\Lambda 0}$ as free parameters in (13), with the other two set to 0, it was found that $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.32 \pm 0.10, 0.68 \pm 0.10)$ or $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.16 \pm 0.09, 0.84 \pm 0.09)$, depending on the analysis technique used. The results pointed to $\Omega_{\Lambda 0} > 0$ with 3.0σ to 4.0σ confidence. Also, $q_0 < 0$ at 2.8σ to 3.9σ confidence, so the universe appears to be accelerating.

2.4 CMB power spectrum

Further evidence for dark energy lies in the CMB power spectrum. If $\Omega_K = 0$, then the first peak is predicted at $l \approx 200$, which is measured. Since we also know that $\Omega_M 0 \approx 0.3$, we are forced to introduce $\Omega_{\Lambda 0} \approx 0.7$ to be consistent with (7) and $\Omega_{R0} \approx 10^{-4}$. This is consistent with the supernova results.

Any non-dark energy explanation for the supernova results must also explain the CMB power spectrum in order to be viable. However, the CMB predictions are based on the cold dark matter model for structure formation, while the supernova results are based on general relativity and the homogeneity and isotropy of space. So the theory behind the CMB results may not be considered quite as strong as the supernova theory, although it is widely accepted.

2.5 Calculating ρ_Λ

It is thought that dark energy may be the zero-point energy from quantum field theory. If this is the case, then we can try to calculate ρ_Λ. It will be the sum of the components from each of the various fields, some of which may be positive and some negative. A sample calculation in [3] shows that for a massless real bosonic scalar field, we get an energy density bigger than 100 g/cm^3, which is much larger than the observed ρ_Λ.

The only values we would therefore expect to calculate for the dark energy density would be zero or some huge number. A huge number is not reasonable on physical grounds. So we really would only expect to get zero. The very small value that we do get, 10^{-29} g/cm^3, seems improbable. This is one reason why it would be nicer theoretically if there was no dark energy.

3 Summary of paper

In the paper I selected [2], E. Barausse, S. Matarrese, and A. Riotto attempt to provide an explanation for the apparent acceleration of the universe by considering the effect of inhomogeneities on the luminosity distance–redshift relation (13).

They first compute the d_L–z relation to second order in perturbation theory in general, and then for a specific metric which contains small perturbations to a background flat matter-dominated universe. An expansion about zero redshift is then performed as in (16) and the measurable values of q_0 and H_0 are calculated in terms of the background values and the perturbations.

The perturbations are not directly measurable, so a range of values must be considered. As a result, $d_L(z)$ goes from being a single-valued function to a multiple-valued function: d_L is not uniquely determined by z. The authors consider super-Hubble perturbations created at inflation, and find that there is a considerable contribution to the variance caused by infrared modes. They discover that the type Ia supernova results fall within the range of possible consequences of a perturbed flat matter-dominated universe.

3.1 Perturbed luminosity distance–redshift relation

The notation in this section follows the notation from [2]. Namely, the metric is written as

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)g_{\mu\nu}dx^\mu dx^\nu \quad , \tag{17}$$

where $\hat{g}_{\mu\nu}$ is the old metric used above, and $g_{\mu\nu}$ is the metric written with the scale factor taken out. Our coordinates now have the conformal time η for the time variable. Quantities written with a ˆ are calculated with the metric $\hat{g}_{\mu\nu}$; otherwise they are calculated with $g_{\mu\nu}$.

3.1.1 General metric

To determine $d_L(z)$ the authors consider the path of a photon $x^\mu(v)$, where v parametrizes the path. The four-momentum of the photon is given by

$$\hat{k}^\mu = \frac{dx^\mu}{dv} \quad , \tag{18}$$

and the energy-momentum tensor for the photon is thus

$$\hat{T}^{\mu\nu} = A^2 k^\mu k^\nu \quad , \tag{19}$$

where A is the amplitude of the wave. By conservation of $\hat{T}^{\mu\nu}$, A evolves as

$$\frac{dA}{dv} = -\frac{1}{2}A\hat{\theta} \quad , \tag{20}$$

where $\hat{\theta} = \hat{\nabla}_\mu \hat{k}^\mu$.

Now working with the metric $g_{\mu\nu}$, with the path of the photon parametrized by λ, we have $d\lambda = a^{-2}dv$ and $k^\mu = a^2\hat{k}^\mu$. It follows, after some work, that

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}k^\mu k^\nu - \frac{\theta^2}{2} - 2|\sigma|^2 \quad , \tag{21}$$

where $\theta = \nabla_\mu k^\mu$ is the *expansion* and σ the *shear* of the congruence along the path $x^\mu(\lambda)$. What these quantities refer to are how much a bundle of photons along a null geodesic spreads out and how much that bundle's shape distorts, respectively (see Appendix F of [4]). We want to determine the amplitude, A and four momentum $\hat{k}^\mu$, of the wave by the time it reaches the observer, and so we need to know how much the photons spread out (see equation (20)). Furthermore, we have that

$$|\sigma|^2 = \frac{1}{2}\left(\nabla_\nu k_\mu \nabla^\nu k^\mu - \frac{\theta^2}{2}\right) \quad , \tag{22}$$

and there is also a differential equation for σ.

Now, an observer with four-velocity $\hat{u}^\mu$ will measure a photon to have frequency $\omega = -\hat{u}_\mu \hat{k}^\mu$, and therefore will measure an energy flux $F = A^2\omega^2$. A source of radius R at position $\lambda = \lambda_s$ along the path (where $\lambda = 0$ is the observer's location), will thus have its total luminosity L given by its surface area times the flux at that position, so $L = 4\pi R^2 l(\lambda_s)$. So by equation (10),

$$d_L = R\left(\frac{l(\lambda_s)}{l(0)}\right)^{1/2} = R\frac{A(\lambda_s)\omega(\lambda_s)}{A(0)\omega(0)} = R\frac{A(\lambda_s)}{A(0)}(1+\tilde{z}(\lambda_s)) \quad , \tag{23}$$

where $\tilde{z}(\lambda)$ is the physical measurable redshift in our perturbed universe.

What remains is to do a perturbation series up to second order to determine d_L. This is done in the standard way. Quantities X such as x^μ, k^μ, θ and σ are expanded as

$$X = X_{(0)} + X_{(1)} + X_{(2)} \quad , \tag{24}$$

and are then substituted into the various equations determined above to calculate the quantities $X_{(i)}$. k^μ and x^μ are first determined by the geodesic equation (since photons follow geodesics), then θ and σ from their differential equations, and finally A from θ. Then from (23) we obtain d_L as a function of λ_s and $\tilde{z}$ (after taking $R \to 0$). After some more work, λ_s can be written in terms of $\tilde{z}$, so we obtain $d_L(\tilde{z})$, and the expression is rather complicated.

3.1.2 Perturbed metric

At this point, a perturbed metric is introduced, of the form

$$ds^2 = -d\eta^2 + \gamma_{ij}dx^i dx^j \quad , \tag{25}$$

where

$$\gamma_{ij} = (1 - 2\psi_{(1)} - \psi_{(2)})\delta_{ij} + \chi_{(1)ij} + \frac{1}{2}\chi_{(2)ij} \quad , \tag{26}$$

where $\chi_{(1)ij}$ and $\chi_{(2)ij}$ are traceless. It is clear that if the perturbations are set to zero, it is simply the RW metric for flat space (because of the $a^2(\eta)$ in (17)). The perturbations can also be expressed in terms of the *peculiar gravitational potential* φ.

The analog of equation (16) is then determined by expanding $d_L(\tilde{z})$ about $\tilde{z} = 0$. The quantities which we would measure through that relation, $\tilde{q}_0$ and $\tilde{H}_0$, are then given in terms of the values in the background flat matter-dominated universe, $q_0 = 1/2$ and H_0, and the peculiar gravitational potential φ.

$$\tilde{H}_0 = H_0\left[1 - \left(\frac{1}{18}\nabla^2\varphi - \frac{5}{108}(\nabla\varphi)^2 + \frac{5}{27}\varphi\nabla^2\varphi\right)\left(\frac{2}{H_0}\right)^2 - \left(\frac{1}{189}\partial^i\partial^j\varphi\partial_i\partial_j\varphi + \frac{1}{252}(\nabla^2\varphi)^2\right)\left(\frac{2}{H_0}\right)^4\right] \quad , \tag{27}$$

and

$$\tilde{q}_0 = \frac{1}{2}\left[1 + \left(\frac{5}{18}\nabla^2\varphi + \frac{25}{27}\varphi\nabla^2\varphi - \frac{25}{108}(\nabla\varphi)^2\right)\left(\frac{2}{H_0}\right)^2 - \left(\frac{1}{30}(\nabla^2\varphi)^2 + \frac{23}{270}\partial^i\partial^j\varphi\partial_i\partial_j\varphi\right)\left(\frac{2}{H_0}\right)^4\right] \quad , \tag{28}$$

where φ is evaluated at the observer's position.

The quantity φ is not an observable quantity, so we have to estimate some range in which it could be found, and then see what sort of variance that leads to in $\tilde{q}_0$ and $\tilde{H}_0$.

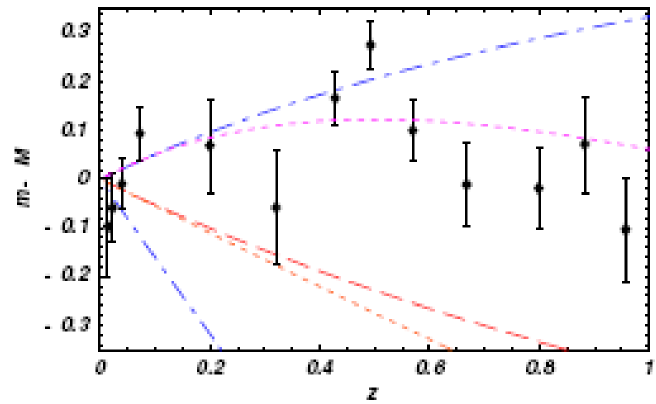

Figure 1: $(m-M)$ versus z for type Ia supernovae. $(m-M)$ is related to the luminosity distance d_L. The area within the blue lines refers to a variance of 1 about $q_0 = 1/2$. The pink line is $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.3, 0.7)$ red is flat matter-dominated ($q = 1/2$) up to $O(z^3)$, and orange is unperturbed flat matter-dominated. Note that most supernovae lie between the blue lines. Source: [2].

3.2 Consequences of super-Hubble perturbations

The gravitational potential φ, caused by perturbations, has a mean of zero and is distributed in a Gaussian way around that mean. So $\tilde{q}_0$ has a mean of $q_0 = 1/2$ and a variance distributed about that value. The paper relates the variation in φ to averages of modes $\vec{k}$ of inflationary perturbations. After some work, it is shown that the variance in $\tilde{q}_0$ may be written

$$\frac{\sqrt{\mathrm{Var}(\tilde{q}_0)}}{q_0} \approx 10^{-10} \ln \frac{k_{MAX}}{k_{MIN}} \quad , \tag{29}$$

where k_{MAX} and k_{MIN} are cutoff values in this averaging procedure. This variance is sensitive to the infrared cutoff k_{MIN}.

The above ratio is given by

$$\frac{k_{MAX}}{k_{MIN}} \approx 10^{-30} \left(\frac{T_{RH}}{T_I}\right) e^N \quad , \tag{30}$$

where T_{RH} is the re-heating temperature after inflation, H_I is the Hubble parameter during inflation, and N is the number of e-folds by which the universe expanded during inflation.

The measured value of $\tilde{q}_0$ is, by (8), $\tilde{q}_0 \approx 0.3/2 - 0.7 = -0.55$. To achieve that value, given a background value $q_0 = 1/2$ for a flat matter-dominated universe would require a variance in $\tilde{q}_0$ of roughly 1. By (29) and (30), this would amount to $N \approx 10^{18.8}$ for a Harrison-Zel'dovich ($n_s = 1$) spectrum, or $N \approx 700$ for $n_s = 0.94$. $N \approx 10^{18.8}$ seems unreasonably high, but $N \approx 700$ is possible. WMAP 3-year results say that $n_s \approx 0.95$, so this seems reasonable. Supposing this variance in $\tilde{q}_0$ is equal to 1, we see in Figure 1 that the type Ia supernova data are consistent with a $q_0 = 1/2$ background.

3.3 Conclusions

The authors of [2] seem to have found an explanation for why we observe the universe to be accelerating. They considered super-Hubble perturbations from a statistical point of view, and found that there was a reasonable chance that they are the cause of our locally observed acceleration. On a larger scale though, the universe could still be flat and matter-dominated.

It would be nice to investigate the CMB power spectrum with a new angular diameter distance equation similar to the new luminosity distance equation to determine whether our observations of the CMB are also consistent with a flat-matter dominated universe. However, the expressions (27) and (28) only hold for small values of $\tilde{z}$, so more work must be done. We also have the issue that, by other methods, we know that $\Omega_{M0} \approx 0.3$, so if the universe is matter-dominated then it cannot be flat. [2] refers to a universe which is flat and matter dominated with $\Omega_{M0} \approx 0.3$ at one point, which did not make complete sense to me.

4 Response

The Barausse, Matarrese and Riotto paper was released around the same time as a paper by Kolb, Matarrese, Notari and Riotto [6], which also concerns super-Hubble perturbations as an explanation for the apparent acceleration of the universe. They solve Einstein's equation for a metric

$$ds^2 = -dt^2 + a^2(t)e^{-2\Psi_l(t)}\delta_{ij}dx^i dx^j \quad , \tag{31}$$

where $\Psi_l(t)$ are time-dependent super-Hubble perturbations created during inflation. The same results as [2] are obtained: that a local observer might determine the universe to be accelerating, even though it is flat and matter-dominated. It is argued that these super-Hubble modes will "mimic" a dark energy fluid.

4.1 Criticism

Both [2] and [6] were heavily criticized shortly after being published. Geshnizjani, Chung and Afshordi [7] point out that the perturbed metric given by (25) and (26) is really the same as the standard RW metric (1), where the perturbation is equivalent to the curvature, and that the effect of perturbations is really quite small. Hirata and Seljak [8] show that the large contribution from infrared modes to the variance in $\tilde{q}_0$ is cancelled by terms that were omitted in the analysis by Barausse et al. Finally, Flanagan [9] also argues that the effect of super-Hubble perturbations is small and presents a simple causality argument against such a solution.

4.1.1 Perturbations amount to renormalization of spatial curvature

In [7] it is shown that for small values of the curvature, k, we can express it in terms of the gravitational potential φ from the Barausse et al. paper as

$$k = \frac{10}{9}\nabla^2\varphi + O[\phi_0^2\nabla^2\varphi, (\nabla^2\varphi)^2] \quad . \tag{32}$$

The result (28) may then be obtained by using the expression (9) for q_0 in a matter-dominated universe.

It is also true that curvature cannot cause q_0 to become negative, so if these perturbations are the same as curvature, they can never cause $q_0 < 0$. The authors then constrain the variance in q_0 to be less than 0.02 from WMAP and less than 10^{-10} from inflation.

4.1.2 Causality argument

Super-Hubble perturbations, by definition, are of size larger than the Hubble radius. On the other hand, the type Ia supernova study looked at $z < 1$. The spacelike hypersurface, Σ, which is bounded by what we observe at $z = 1$ is much smaller than the Hubble radius, so over that region super-Hubble perturbations are hardly noticeable. But anything outside Σ, by causality, can have no effect on objects we see at $z < 1$. In particular, the only part of the super-Hubble perturbations that could have an effect is contained within Σ, and thus essentially constant. In fact, we know from observations that within Σ, perturbations are of order $\delta\rho/\rho \approx 10^{-4}$. It is hard to imagine how this could cause local acceleration.

4.2 Related ideas

While many flaws have been found in [2] and [6], and super-Hubble perturbations seem unlikely to be able to cause local acceleration, the idea of providing alternatives to dark energy through inhomogeneities has continued to be pursued.

4.2.1 Sub-Hubble perturbations

After the super-Hubble perturbations paper, Kolb, Matarrese and Riotto released a paper dealing with sub-Hubble perturbations and their effect on local acceleration [10]. This paper took a different approach. Whereas the Barausse et al. paper deals directly with observable quantities (the d_L–z relation), the newer Kolb et al. paper defines new cosmological parameters which are spatially averaged over large regions. The authors argue that what we really measure are these coarse-grained quantities.

Over a spatial domain D, coarse-grained values of quantities are defined. For example, the coarse grained value of a quantity F is given by

$$\langle F\rangle_D = \frac{\int_D F\sqrt{h}d^3x}{\int_D \sqrt{h}d^3x} \quad , \tag{33}$$

where $h_{\mu\nu}$ is the spatial part of the metric.

An important fact is that coarse-graining and time differentiation do not commute, which follows from the non-linearity of Einstein's equation. As a result, the Friedmann equations for these coarse-grained quantities are different from the Friedmann equations for the bare quantities. In the new Friedmann equations, ρ and p are replaced by the *effective* energy density and pressure, ρ_{eff} and p_{eff}, which depend on the bare values as well as the expansion θ and shear σ.

Using a perturbed metric and the new Friedmann equations, it was shown that second order effects of sub-Hubble perturbations can lead to acceleration of the coarse-grained scale factor.

This paper was criticized by Wald and Ishibashi [11] who argued that these coarse-grained quantities are in fact not what is actually measured. They gave an example of a universe consisting of two disconnected matter-dominated regions, one of which is increasing in size; the other one decreasing. Since they are matter-dominated, $q > 0$, so they are decelerating. However, the quantity a_D, which is the coarse-grained scale factor, is accelerating. So all observers would measure deceleration, but $\ddot{a}_D > 0$.

Wald and Ishibashi also pointed out that what we observe of space is very nearly homogeneous, and thus well described by the Newtonionly perturbed RW metric,

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Psi)\gamma_{ij}dx^i dx^j \quad . \tag{34}$$

In almost all cases, $\Psi < 10^{-5}$, so any correction terms to the Friedmann equations from perturbations should be negligibly small, so it is unlikely that second order effects could ever account for our observed acceleration.

4.2.2 Lemaître-Tolman-Bondi inhomogenous spacetimes

Rather than perturb an RW metric with inflationary perturbations, some authors have decided to assume that we are in a locally underdense region [12, 13]. If the universe is isotropic, but not homogeneous, then it is described by the Lemaître-Tolman-Bondi metric,

$$ds^2 = -dt^2 + X^2(r,t)dr^2 + R^2(r,t)d\Omega^2 \quad . \tag{35}$$

In the paper by Alnes et al. they consider the Earth to be near the middle of an underdense bubble, and they look at the effect of this on the d_L-z relation and the prediction for the position of the first peak in the CMB power spectrum. This is similar to the Barausse et al. paper in that they are looking at actual observable quantities, unlike the sub-Hubble Kolb et al. paper.

With a homogeneous universe, q is a function of time only, but in an inhomogeneous universe, it is a function of space as well,

$$d_L(z) = (1 + z)^2 R(r,t) \quad . \tag{36}$$

This means that while previously we had to attribute the supernova results to a universe which is accelerating everywhere, now they may be explained by spatial variations in the expansion rate of the universe. The Garfinkle paper shows that with appropriate choices of parameters, the supernova data fits the underdense bubble theory very well.

When applied to the CMB, the model by Alnes et al. successully predicts the location of the first peak in the temperature power spectrum. The other peaks have not been looked at though.

5 Conclusions

If we assume that our universe is homogeneous and isotropic, then observations of the flux from type Ia supernovae at known redshifts imply that the universe is accelerating and that we must have either a cosmological constant or a dark energy fluid with $w \approx -1$. The dark energy density is very small, and hard to predict theoretically. Therefore, it would be good if we could account for the supernova measurements by supposing the universe is not quite homogeneous.

The paper by Barausse et al. attempted to do this by calculating a modified luminosity distance–redshift relation for a perturbed universe, and then examining the effect of super-Hubble perturbations on it. Because the perturbations are not known, a range of perturbations must be considered, so that we have a range of values of luminosity distance for a given redshift. The super-Hubble perturbations were shown to lead to a big enough variance to accommodate the supernova observations. However, numerous other authors have criticized this paper and it seems to be incorrect.

The idea of using inhomogeneities to explain the observed acceleration of the universe is still being examined, but the techniques have changed. Sub-Hubble perturbations and underdense bubbles are more popular now. To be successful, a new model must successfully account for the CMB power spectrum peak locations as well as the supernova observations. Since the existence of dark energy poses new theoretical problems, these models which attempt to eliminate our need for it should be fully investigated.

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