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Big Bang Models in Matrix String Theory

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Abstract

In this paper we review a recent proposal, called the Matrix Big Bang, to model the big bang using matrix string theory. We start with a brief review of singularities and the required concepts from string/M-theory, focussing on concepts and omitting technical details. The Matrix Big Bang is then described qualitatively, highlighting its major points. Finally, a brief discussion of how the Matrix Big Bang fits into the standard model of cosmology is given. We find that while there are no immediate conflicts with the present cosmological lore, there are limited observational consequences. Indeed, the major success of this model is its theoretical framework: it is one of the first concrete realizations of pre-Planck time physics.

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1 Introduction

General relativity is a masterpiece of classical (non-quantum) physics. It has many well tested features and predictions, and provides an excellent description of the cosmology of the universe. Yet, it cannot answer some of the most basic questions: what is the beginning of time? What was before the big bang, if anything? These questions revolve around the issue of singularities: a region in spacetime where classical general relativity breaks down. Two of the most prominent examples of singularities are black holes and the big bang. In classical physics, these singularities are poorly understood, as near the singularities quantum effects become important. This motivates one to search for a quantum theory of gravity. The leading candidate is string theory, which is fundamentally quantum mechanical, and predicts a spin 2 graviton: it is a consistent finite theory of quantum gravity. Moreover, string theory has a well known description of black hole physics which is *non-singular*. In fact, many important non-perturbative effects of string theory were discovered by studying black hole singularities. On the other hand, a consistent description of timelike singularities, including the big bang, have remained elusive. One reason for this is that cosmological backgrounds (metrics) are time dependent, and are hard to study in string theory. In contrast, blackhole singularities have static backgrounds which are more amenable to string theory techniques. However, if string theory is truly a theory of quantum gravity then it should provide a clean description of physics both near and *at* the big bang. Moreover, just as studying black holes facilitated many fruitful ideas in string theory, studying cosmological singularities will hopefully produce similar new ideas and insights.

From a cosmological perspective it is strongly desired to have an understanding of the big bang, as this is a gaping hole in the standard model of cosmology. Any resolution of this hole in any context, including string theory, would be a major step forward for the consistency of cosmology.

In this paper we will review one such proposed solution, termed a 'Matrix Big Bang' [1], [2]. It is a non-perturbative description of the universe in the context of string theory, or rather M-theory (which subsumes string theory). In section 2, we will review and motivate the study of cosmological singularities. The elements of string theory needed to described this model are also briefly reviewed at a non-technical level. In section 3 we overview the Matrix Big Bang describing their essential features. In section 4 we discuss how these models fit into the standard model of cosmology, and possible observational and theoretical consequences. Finally in section 5 we give a summary and conclusion.

2 Singularties in General Relativity and String Theory

Two of the most prominent examples of singularities in cosmology are the big bang and black holes. Black holes are well known static singularities, which string theory has had some success in describing (see [3] for a review). The big bang, on the other hand, is more important to cosmological models and is discussed hereon in.

The Friedmann-Roberston-Walker spacetime in spherical polar coordinates is given by

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1}$$

where k is the curvature of the spatial sections. In all of the following we will consider flat spacetime, where $k = 0$. In the FRW metric the scale factor $a(t) \to 0$ at some finite time implying a singularity where spacetime is infinitely curved. This implies a breakdown in the description of the physics given to us by general relativity: the theory is missing UV degrees of freedom, in particular quantum effects. The general reason for this is as follows. It is well known that general relativity cannot be quantized as a field theory: it is not renormalisable, meaning that it suffers from intractably many divergences. However, it is now thought that general relativity is an effective theory of gravity, being the low energy limit of string theory. In particular, near such singularities stringy effects become important and result in smearing the singularity giving finite results. String perturbation theory describes gravitons (quanta of gravity) as oscillating closed strings instead of being localised point particles. It is this non-locality that results in the above smearing of the singularity and indeed of all traditional point-like interactions. This results in a finite theory of gravity.

String theory has been very successful in solving UV finiteness problems, and also in resolving spacelike singularities like black holes. However, string theory in isolation has been unsuccessful in describing cosmological singularities. The basic reason for this is that gravitons always see an effective Newton's constant that is diverging $G_N \to \infty$. A related problem, as discussed below, is that string theory has a 'coupling constant' g_s, which must be small for perturbative string theory techniques to apply. Near the big bang, $g_s \to \infty$, implying that string perturbation theory is not a useful description of the physics. Thus it

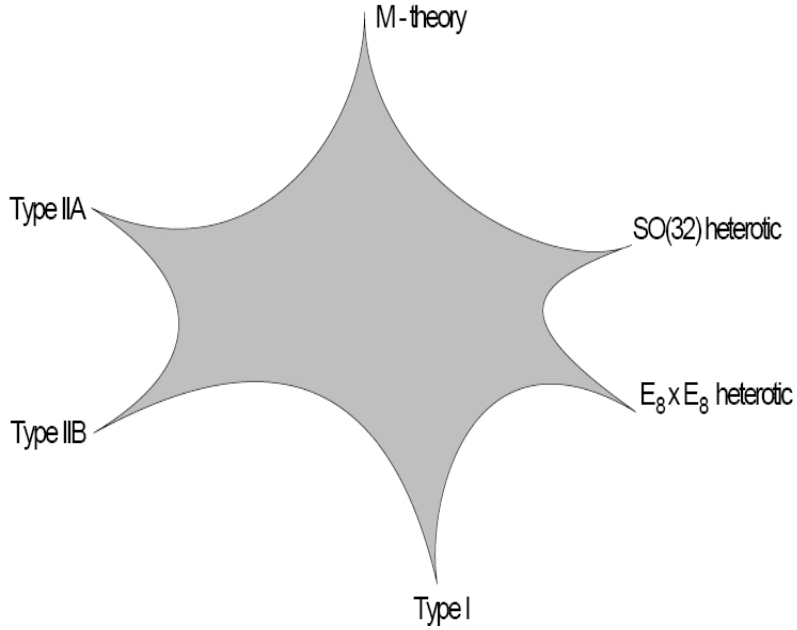

Figure 1: The web of M-theory. Each of the five consistent string theories are limits of a single master theory called M-theory. Image from [6].

is necessary to have a better theory. The proposal of [1], [2] is to use M-theory. Thus, let us now turn to a brief discussion of M-theory and illustrate the relevant concepts.

2.1 A brief overview of M-theory

Until the mid 1990s it was thought that there were five known consistent string theories, labelled type IIA, IIB, Heterotic E, Heterotic O, and type I. Each of these strings theories are mathematically consistent only in 10 dimensions. During 1995-1999 however it was discovered that in fact these are particular limits of an 11-dimensional theory, dubbed M-theory. This is shown in figure 1. In particular, M-theory is non-perturbative in that there is no small parameter (like $\hbar$) in which to make a perturbative expansion. This is in contrast with more familiar field theories like QCD, which have a small coupling constant in which one can expand perturbatively. Even in string theory, there is a coupling constant g_s, which when small means one can study string perturbation theory. In addition to this, it is difficult to find a completely general definition of M-theory. Nevertheless, one can make some progress. It is possible to write down a low energy approximation to M-theory via supergravity (SUGRA). This comes in the form of a 11d metric. Moreover, four authors, Banks-Fischler-Schenker-Susskind, conjectured a non-perturbative definition of M-theory in terms of D-branes, which are a type of non-perturbative object in string theory [4]. In the next section we describe D-branes and their relevance to the BFSS conjecture. For a review of M-theory see [5] or as it applies to cosmology see [6].

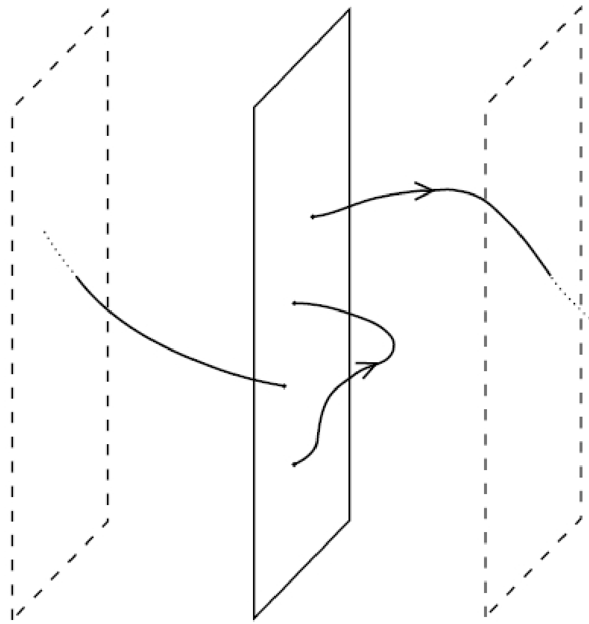

Figure 2: A cartoon showing the idea of a D-brane. The brane is a surface upon which open strings may end. An open string can stretch between two adjacent D-branes or end on the same D-brane. Image from [7]
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Let us now briefly describe the relevant concepts of D-branes.

2.2 D-branes

It is well known in cosmology that one can have topological defects, for example domain walls or cosmic strings. D-branes are an example of such an object in string theory. A Dp-brane is a $p + 1$ dimensional surface embedded in a higher dimensional spacetime (10 dimensions for string theory) upon which open strings may end as shown in figure 2. Just as domain walls are solitonic solutions so are D-branes. Consequently, they are non-perturbative: they do not appear at any order in string perturbation theory. For some reviews of D-branes in string theory see [7] and [8].

The degrees of freedom on D-branes are excitations of the attached open strings, and these result in an important concept: one has an effective field theory 'living' on the brane. That is, if one imagines an observer living on surface of the Dp-brane, then its effective world is $p+1$ dimensional. The effective field theory is what these attached strings would look like from this $p+1$ dimensional world. In particular, the field theory is very familiar: it is supersymmetric Yang-Mills (SYM). The field content of SYM consists of a gauge field A^μ, and Lorentz scalars X^i, where i is an internal index. The interpretation of A^μ is a gauge boson (eg. a photon or W boson) on living on the brane, while the scalars X^i describe the position of the D-brane in the higher dimensional spacetime.

With N D-branes additional symmetries can exist. In particular, open strings may stretch between any two D-branes resulting in additional degrees of freedom on the brane. These additional degrees of freedom result in a non-abelian gauge theory where the fields are now non-commuting matrices. The scalar fields X^i, which are now $N \times N$ matrices have a potential term

$$V \approx \mathrm{tr}\,[X^i, X^j]^2. \qquad (2)$$

The diagonal elements of X describe the position of the D-brane, and as diagonal matrices commute these remain massless. These modes are termed *flat directions*, and there are N such degrees of freedom. With separated D-branes, connected strings become stretched resulting in the off-diagonal elements getting a mass. The massive degrees of freedom are typically truncated, leaving N degrees of freedom in total. For example, in section 3 the massless degrees of freedom will correspond to our ordinary concept of N gravitons. When two D-branes coincide, the stretched strings become massless, and one has a point of 'enhanced' symmetry, where there are additional massless degrees of freedom from the off-diagonal elements. This results in N^2 degrees of freedom, and the gauge theory becomes non-abelian $U(N)$ SYM. This idea will be important in section 3 when we discuss the physics near the big bang. There the N gravitons will be replaced by N^2 non-abelian degrees of freedom – a concept far from our ordinary intuition.

The relevant examples of D-branes for us will be D0-branes and D1-branes. D0-branes are point particles, and hence described by $0+1$ dimensional field theory, that is quantum mechanics! When truncated to the massless degrees of freedom, the field content is relatively simple: there are 9 scalars X^i, $i = 1, \ldots, 9$ describing the position of the D0-brane, and their fermionic superpartners. Note that there are no physical gauge degrees of freedom. As described above, when there are multiple coincident D0-branes the quantum mechanics is promoted to a non-abelian quantum mechanics with X's becoming matrices. This is a hint that spacetime may look fuzzy or non-commutative. The D1-branes have one extended spatial direction and so look like strings (they are sometimes called D-strings). The field content consists of 8 scalars, a gauge field and their fermionic superpartners. D1-branes will be useful in describing 'spacetime' near the big bang.

Now that we have all of the relevant concepts, we can return to the subject of M-theory, and for us, the relevant definition: BFSS.

2.3 BFSS and Matrix Theory

Let us now discuss the critical ingredient to the Matrix Big Bang model: the BFSS conjecture. As mentioned above, M-theory has proved difficult to define. For example, one major difficulty is in finding a background independent definition: one that is independent of the background spacetime metric. However, if one ignores this problem then there do exist non-perturbative definitions, and the BFSS conjecture is one of those. Broadly speaking, the BFSS conjecture states that M-theory is equivalent to type IIA string theory with N D0-branes. One must take a 'decoupling' limit after which the D0-branes represent gravitons, and can undergo scattering processes and other such S-matrix interactions. This combination of processes is often referred to as Discrete Lightcone Quantisation (DLCQ). As described in the previous section, the D0-branes interact via attached strings, and in the decoupling limit we consider only the massless degrees of freedom. Then, when the D0-branes are well separated, they look like N individual gravitons and only the massless diagonal degrees of freedom contribute. The massive off-diagonal degrees of freedom are suppressed in the decoupling limit. When the D0-branes come together to interact, the off-diagonal degrees of freedom become light, and we enter a non-abelian gluon phase, where the interacting field theory is non-abelian. This is a distinct departure from conventional theories of gravity, but is critical to the understanding of cosmological singularities in the Matrix Big Bang model. In the next section, we will elaborate on this non-abelian phase further in the context of the Matrix Big Bang.

Now that we have a relevant definition of M-theory as a matrix theory, we turn to discussing the Matrix Big Bang proposed in [1], [2]. A closely related proposal has also been given in [10], [9].

3 The Matrix Big Bang

In this section we describe the Matrix Big Bang, putting together some of the concepts reviewed above. We focus on two papers: [1] and [2]. The former establishes the Matrix Big Bang, in particular discussing its kinematic features, while the latter examines dynamical effects resulting from quantum corrections. Firstly however, it is necessary to introduce a time-dependent background, analogous to the FRW metric (1) above.

Flat Minkowski space is a common, simple background for field theory, and string theory but it is static (independent of time). On the other hand, the FRW metric (1) is inherently time dependent. The analogue of an FRW metric in string/M-theory is generally hard to reproduce. However, there are close analogies, where the time dependence is given by *null time*. This is the background studied in the Matrix Big Bang, and it is useful as it facilitates significant simplifications in solving the M-theory equations of motion. Null or lightcone coordinates are given by

$$X^{\pm} = \frac{1}{\sqrt{2}} (t \pm x) \,, \tag{3}$$

where x is some spatial direction and t is time. One should interpret X^+ as being null time, and X^- as a spatial direction[2]. In lightcone coordinates, the Minkowski metric becomes

$$ds^2 = -2dX^+dX^- + (dX^i)^2, \tag{4}$$

where $i = 2, \ldots, 9$, for a type II background.

The backgrounds discussed in Matrix Big Bang in [1], [2], have the metric (4) above, while a null-time dependence is induced via the string coupling g_s

$$g_s = e^{-QX^+}, \tag{5}$$

where $Q \neq 0$ is a parameter whose value is of no physical significance. It is shown in [1] that it is possible to define string pertubation theory using a lightcone formalism similar to the coordinate choice (3) above. As shown below, in the late time limit this perturbative description will emerge [3], implying conventional space and time are emergent from the big bang.

There is a subtlety here, however. The background (4) is what appears in string theory. However, it turns out that in writing down the low energy limit of string theory, the Ricci scalar appears in a non-standard form. Therefore, in order to compare with conventional physics one needs to convert to the 'Einstein' frame by performing a conformal transformation viz. [11]

$$\begin{aligned} ds^2 &= g_s^{-1/2}(-2dX^+dX^- + (dX^i)^2), \\ g_s &= e^{-QX^+}. \end{aligned} \tag{6}$$

This is now a curved metric, and it is in this reference frame that we can compare with conventional general relativity. The big bang corresponds to early times $X^+ \to -\infty$, and

[2]Lightcone coordinates parametrise the path of a lightray in the x direction, hence the name 'lightcone'.

[3]Note that it is perturbative type II string theory that describes gravitons as oscillating strings and hence 'conventional' space and time.

from (6) we see that there is a singularity where the scale factor is shrinking to zero size and the string coupling is diverging. This is reminiscent of the singularity described in the introduction, where the FRW metric (1) exhibited a similar singularity. Moreover, the fact that the theory is strongly coupled implies that conventional perturbative techniques break down. This is again reminiscent of general relativity breaking down in the FRW context. However, the power of non-perturbative string theory now comes to the fore, and in particular duality.

The dual description of this physics comes about from the quantisation of this background. As there is explicit time dependence in the metric (4), the conventional DLCQ method described above does not work. However, as shown in [1], [2] there is a closely analogous procedure, which when naively followed through yields D0-branes. For technical reasons (roughly because it is strong coupled), this is not a good description of the physics. The power of duality can now be used. The D0-branes can be dualised into D1-branes, which are *weakly* coupled and thus a good description of the physics near the big bang. That is, the dynamics of the D1-branes is described by a $1 + 1$ dimensional effective field theory with coupling g_{YM}, where

$$g_{YM} \sim \frac{1}{g_s}. \tag{7}$$

This is known as matrix string theory [12]. Thus at early times around the big bang $X^+ \to -\infty$, the dual matrix string description is weakly coupled, and can be analysed using conventional perturbation theory. This is a highly non-trivial and crucial statement. To reiterate: the region of strong coupling in the original IIA background (4),(5) is described by a weakly coupled D1-brane description. It is this observation that allows one to get control over the singularity at the big bang. Let us analyse the behaviour of this theory in more detail.

The matrix string field theory has a potential given by

$$V \sim g_{YM}^2 \mathrm{tr}\, [X^i, X^j]^2, \tag{8}$$

which at early times is small. That is, the off diagonal elements are massless, and this corresponds to a non-abelian gauge theory with N^2 light degrees of freedom. Thus the big bang is described by a non-abelian phase, which as discussed in section 2.2 can be pictured as N D1-branes coinciding. This is illustrated in figure 3. This non-abelian phase, as argued in [1], should replace our notion of spacetime near the big bang. Moreover, at late times $Q^+ \to \infty$, the 1+1 field theory becomes strongly coupled, implying the

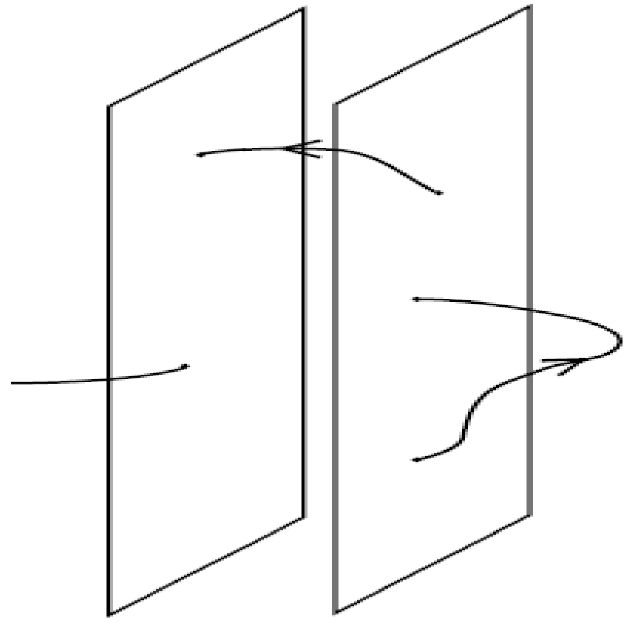

Figure 3: A figure showing two D-branes close to coinciding. The stretched string is becoming lighter as the separation is decreased and its tension decreases correspondingly. Image from [6].

matrix string description is breaking down. However as shown in [12], only the N flat directions contribute and matrix string theory in this limit reduces to the perturbative IIA string theory description. Hence we recover our conventional notion of gravitons and hence spacetime.

One could ask the question, 'does time begin?' Indeed, this has been asked in the context of similar singularities, for example whether gravitons can scatter through a Milne singularity in perturbative string theory [13], [14]. It was found there that the perturbative calculation diverged at the lowest order, due to a large gravitational backreaction. That is, there was a large feedback response (imagine a microphone next to a loudspeaker). In this model however, spacetime is described by D-strings and not by perturbative strings so this backreaction should be absent at best, or milder at least. However, this remains an open question as acknowledged by [1], as their model deals with time after the big bang.

To summarise: the big bang is described at early times by a 1+1 weakly coupled matrix field theory on D1-branes. At late times it is argued that the matrix degrees of freedom re-organise themselves into conventional strings: perturbative string theory describes this universe at late times. That is, conventional spacetime emerges at late times.

3.1 Dynamical Effects

The discussion in the previous section has focussed primarily on classical and non-dynamical effects. The thrust of [2] is to study the dynamical effects (that is quantum effects) resulting from higher order corrections in the field theory. The calculations performed in [2] are largely technical in nature and not reproduced here. We state some of the results in a qualitative nature only and the interested reader is referred to the original paper.

Firstly, one should note that a type of symmetry called supersymmetry[4] is completely broken, resulting in the generation of an effective potential. Calculating this effective potential is a non-trivial exercise, as the background (6) is time-dependent. In other similar situations in string theory and M-theory, the presence of this effective potential will often lift the flat directions at late times. That is, the N diagonal degrees of freedom will acquire a mass dynamically when quantum effects are taken into account. However, this does not happen in this case: quantum effects imply the flat directions remain flat at late times. This is reassuring, as one wants to recover conventional spacetime as discussed in the previous section.

There is a characteristic time scale given by the time when the matrix string description breaks down. This occurs when $g_{YM}/b \sim 1$, where b is the characteristic mass scale of the off diagonal degrees of freedom. This gives a time scale

$$\tau_{string} \sim \frac{1}{Q} \ln(l_s b), \tag{9}$$

where l_s is the typical string length.

For early times near the big bang, the effective potential turns off, and the theory becomes effectively free. That is, the dynamics are as described in the previous subsection. For late times, of the order τ_{string}, the flat directions are restored quantum mechanically as mentioned above. This is due to the large suppression of the potential at late times. This is understood as the restoration of supersymmetry at late times. Again this is in accordance with the emergence of spacetime.

This concludes the two papers. Let us briefly summarise what we have discussed so far:

- the resolution of the singularity at the big bang is a large hole in the overall cosmological framework;
- the approach of the Matrix Big Bang is to control the big bang singularity using the non-perturbative framework of matrix theory;

[4]Supersymmetry says that for every boson there is a corresponding fermion, and the pair together are called superpartners. Supersymmetry often implies very strong constraints on quantum corrections, and in many cases will result in quantum corrections cancelling. This is not the case in this model however.

- the big bang corresponds to a non-abelian phase described by D1-branes;
- conventional spacetime emerges at late times in accordance with observations;
- dynamical effects support the above conclusions.

Let us now turn to a brief discussion of the cosmological consequences of these models, as well as possible future directions one could take.

4 Cosmological Consequences and Future Directions?

As emphasized in the previous two sections[5], the cosmological success and consequences of this model are primarily theoretical. Planckian physics is a huge unknown in the standard model of cosmology. However, the Matrix Big Bang is an example of an attempt by string/M-theory to fill this void. With a few exceptions, for example [10], [9], this is one of the only concrete realizations of cosmological singularities in the framework of string theory, or indeed, quantum gravity. This in itself makes the model cosmologically interesting.

Now one could ask the obvious question: what are the cosmological consequences and observables, if any? The is currently an open question. Let us analyse this point somewhat. Indeed, as discussed in the previous section, post-Planckian physics results in conventional spacetime emerging. What are the observables that emerge from this model? Recall from section 3 that at late times the universe that emerges is described by type IIA string pertubation theory. Is type IIA a reasonable description of the universe as we know it in standard cosmology? Or in a more general context, suppose a generic string theory was emergent: is string/M-theory as a whole a good phenomenological description of the universe as we know it?

The answer is currently hotly debated. Aside from mathematical consistency, string theory has phenomenological predictions that are worthy of note: a spin 2 particle, the graviton, as well as the photon and fermions. However, there are many unobserved particles and untestable predictions, and in particular type IIA is not known as being a good candidate for phenomenology. Thus the Matrix Big Bang described above, suffers from many of the problems common to string theory generically. Or in other words, the observational predictions of this model come down to that of string theory: any confirmation that string theory and especially type IIA are correct would be evidence for this model.

[5]Note that, in this next section we make some brief comments on the papers and how they fit into modern cosmology. These are generally not from the papers themselves.

Nevertheless string theory is the leading candidate for a quantum theory of gravity. Indeed, the Matrix Big Bang is an example of where string theory succeeds in places that conventional physics, viz. general relativity, has failed. We have seen that the Matrix Big Bang has control over the physics at early times near the big bang, and moreover describes what spacetime becomes at the big bang: a non-abelian phase of matrix strings. Moreover, this model is aimed more towards establishing a paradigm or framework for handling cosmological singularities. Its aim is not at creating completely correct phenomenology, which is an open and challenging question for future direction. Moreover, just as studying string theory near black hole singularities yielded many fruitful ideas, one would hope that studying string/M-theory near cosmological singularities would lead to many more fruitful ideas. Consequently, one should not judge the model by its phenomenology but rather by its theoretical progress, which is significant.

There are a number of directions the papers [1] and [2] could take. One technical direction would be to extend the effective potential calculated above to higher orders in perturbation theory. This would have consequences on the late time predictions such as whether the flat directions remain massless or not. Another challenging direction could be to develop models that evolve in real time. Lightcone time results in many simplifications, but is ultimately unphysical. Further, one could attempt to extend this discussion to other types of string theories, not just type IIA. For example, to those that have realistic phenomenologies. Again, phenomenology is more a restriction of string theory, not of the Matrix Big Bang. As noted in [1] though, it would be interesting to see how this model would evolve given some initial state. It would presumably emerge into some configuration of gravitons and other particles, but what would they be? This would be a step in the direction of making concrete observational predictions, and of obvious relevance to cosmologists. Moreover, there are issues such as what role inflation plays, and are there density perturbations? Are there emergent anisotropies in the CMB? Some of these types of questions in different scenarios have been asked in [15], [16], [17] and such a discussion in the context of the Matrix Big Bang may prove interesting.

5 Conclusion

Finally, let us briefly summarise and conclude. The big bang is a cosmological singularity, which thus far has proven difficult to tame. Classical general relativity fails as it cannot include quantum effects consistently, while string theory to date has also failed for various reasons. The resolution of the big bang is thus a large hole in the cosmological framework.

The missing ingredient as claimed by [1], [2] is matrix string theory. The Matrix Big Bang is a cosmological model that exhibits a big bang type singularity. However, near the big bang, duality allows us to control the singularity using matrix strings. This describes a non-abelian phase, where our concepts of space and time are replaced by D1-branes. At late times however, well after the big bang, we see our usual concept of gravity, space and time emerge, being replaced by type IIA string theory. The model is robust upon including leading quantum effects. There are a number of interesting directions to pursue, as discussed in the previous section.

Although some may doubt the validity of string/M-theory, it is in situations like this that one is reassured as to its power. Although a very specific model, the Matrix Big Bang illustrates that string theory and M-theory are able to control regions of spacetime where classical approaches have failed. One can hope that by such endeavors string theorists will one day be able answer deep questions such as 'what is the beginning of time?'

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