Using the Sachs-Wolfe Effect to Limit Dark Energy

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Abstract

The Sachs-Wolfe effect is a characteristic of the cosmic microwave background (CMB) in which CMB photons are gravitationally redshifted as they propagate outward from the surface of last scattering. There are two categories of Sachs-Wolfe effects: the non-integrated and integrated varieties. The non-integrated SW effect imprints the CMB at the surface of last scattering, while the integrated SW effect causes perturbations in the CMB as the photons travel from the surface of last scattering to the Earth, where they are detected. By correlating CMB temperature anisotropies with other observations, the integrated SW effect can be separated out from the total anisotropy, revealing information about the energy densities the photons encountered on their journey across the universe. Because dark energy makes up a dominant part of the energy density of the universe at large scales, the dark energy changes the photons through the integrated SW effect, and by understanding these changes, we can learn about dark energy. It was found that this process works in principle, though there are several stumbling points, and more measurements on certain other features involved in the experiment could alleviate some of the problems associated with this technique.

1 The Sachs-Wolfe Effect

The Sachs-Wolfe effect is an imprint on the cosmic microwave background (CMB) that results from gravitational potentials shifting the frequency of

CMB photons as they leave the surface of last scattering and are eventually observed on Earth. Two categories of Sachs-Wolfe effects alters the CMB: the non-integrated and integrated types.

1.1 The Non-Integrated Sachs-Wolfe Effect

The non-integrated Sachs-Wolfe effect takes place at the surface of last scattering. The photon frequency shifts result from the photons climbing out of the potential wells at the surface of last scattering created by the energy density in the universe at that point in time. The effect is not constant across the sky due to the perturbations in the the energy density of the universe at the time the CMB was formed. Thus, the Sachs-Wolfe effect makes an imprint on the CMB power spectrum at the time of the last scattering and is therefore a primary anisotropy. The Sachs-Wolfe effect is the primary contributor to variations in the CMB at large angular scales.

1.2 The Integrated Sachs-Wolfe Effect

As the photons pass through the universe on their way to Earth, they can also suffer a second type of Sachs-Wolfe effect, the integrated kind. On their flight, the photons can encounter additional gravitational potentials and gain and lose energy. They gain energy when they descend into a well, and lose it when they climb out. Reasonably, one would expect these changes to cancel out over time, but the wells themselves can evolve, leading to a net change in energy for the photons as they travel. In addition, as the potential wells change, the spacetime itself in which the photons are propagating stretches or contracts, and so the photons gain or lose energy as they are stretched or contracted in wavelength along with spacetime. One can now see why this is the integrated Sachs-Wolfe effect: the effect is integrated over the photon's total passage through the universe. In the end, the power spectrum anisotropies from the non-integrated Sachs-Wolfe effect reveals information about the photons' initial conditions, while the secondary anisotropies from the integrated Sachs-Wolfe effect leaves evidence of the change of space as the photon traveled through it.

The integrated Sachs-Wolfe effect can be further broken down into an early time effect and a late time effect. The early integrated Sachs-Wolfe effect takes place from the time following recombination to the time when radiation is no longer the dominant contributor to the energy density of the universe, but when matters assumes this role. The radiation domination in this epoch means that potentials at the sound horizon scale decay, giving the photons energy. When radiation no longer dominates the universe, the potentials no longer decay and this period comes to an end. The early Sachs-Wolfe effect gives clues about what is happening in the universe at the time when radiation ceases to dominate the energy in the universe.

Just as the early integrated Sachs-Wolfe effect provides information about the end of the radiation-dominated era, the late integrated Sachs-Wolfe effect gives clues about the end of the matter dominated era. When matter eventually gives way to dark energy or curvature, the gravitational potentials decay away. Photons have traveled much farther than they did during the radiation-dominated era, and so are equally likely to be in a region of low matter density as high matter density. During the potential decay, the photons pass over many intervening regions of low and high density, effectively cancelling the late integrated Sachs-Wolfe effect out except at the very largest scales. See Figure 1 for additional discussion of these features. The late integrated Sachs-Wolfe effect will be particularly useful in giving one of the most direct probes of the density parameters Ω_R, Ω_M, Ω_κ, and Ω_Λ.

1.3 Derivation

The Sachs-Wolfe effect can be understood as a conservation of energy, and by changing coordinates of the background the relationship between the fluctuations and the gravitational potential can be derived.

The temperature fluctuations and the gravitational potential are related as follows, for adiabatic fluctuations in a matter-dominated universe:

$$\frac{\Delta T}{T} = -\frac{1}{3}\Phi \tag{1}$$

To get here, we begin with the geodesic equation for a photon propagating in a region with a perturbing potential Φ. We then find:

$$\frac{\Delta T}{T}\mid_f = \frac{\Delta T}{T}\mid_i -\Phi_i \tag{2}$$

The i's and f's refer to initial and final states of the photons, and the Φ_f term has been neglected because it results in a uniform shift, and the Doppler

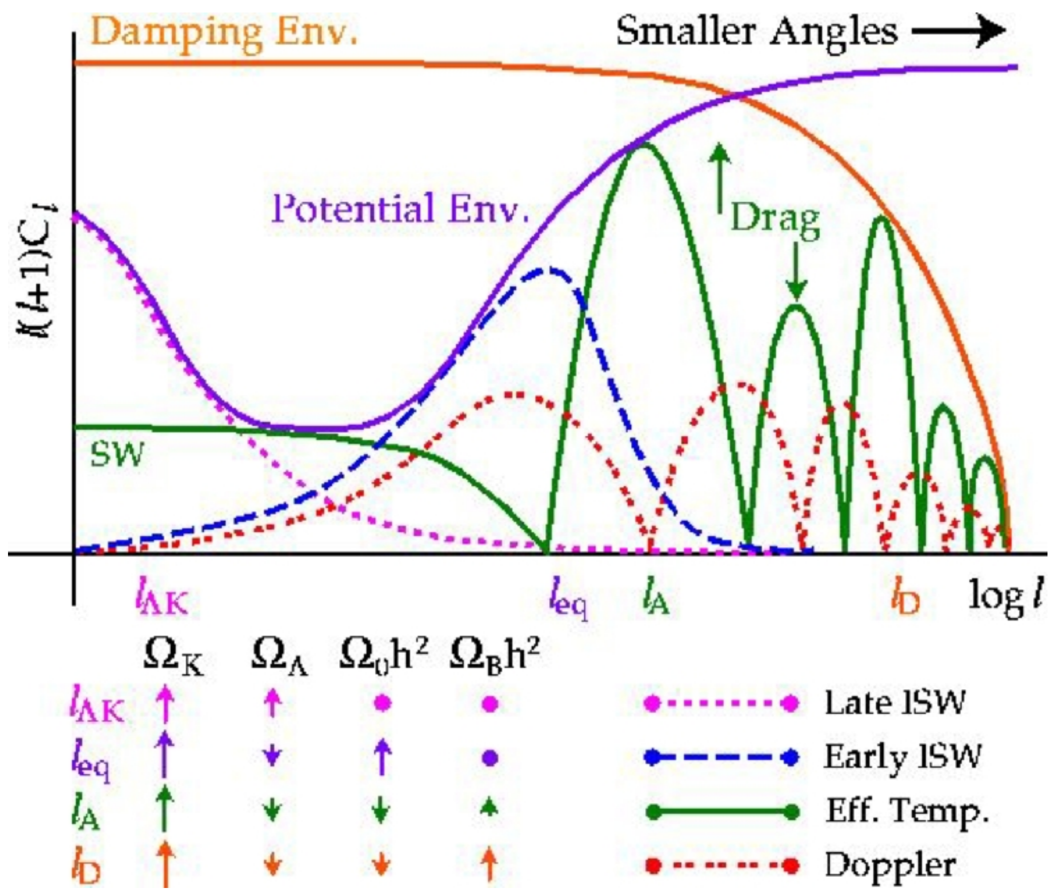

Figure 1: Figure reproduced from Wayne Hu. Note several of the features: the late integrated Sachs-Wolfe effect (pink) line is very small for large ℓ, or small angular size, but grows as ℓ gets small, and the corresponding scale size gets large. The acoustic peaks for the effective temperature (green) line and Doppler (red) lines are also out of phase, as expected. Finally, the effective temperature also shows a boost of the third peak and suppression of the second one. This extra contribution comes from dark matter, which does not couple to photons. The photons try to damp out the acoustic oscillations, but the dark matter reinforces odd peaks (the collapsing portion) and suppresses even peaks (the expanding portion) and the photons do not damp out the dark matter.

terms between emitter and observer have been neglected as small. This equation can be interpreted as a statement of energy conservation. The initial fluctuation term on the right hand side is the inherent perturbation at early times, and the Φ term is the energy lost when the photon leaves the well, leading to the final value of the fluctuation.

Working in the rest frame of the cosmological fluid, and with an appropriate gauge choice, we can find the general expression for the relationship between fluctuations and the gravitational potential. Noting the equation of state is $p = w\rho$, we find the following:

$$\frac{\Delta T}{T} |_i = -\frac{\delta a}{a} = \frac{2}{3(1+w)}\Phi \tag{3}$$

Plugging this expression into the above equation and noting that $w = 0$ for matter, we recover where we began, namely that the fluctuations should go like $-\frac{1}{3}\Phi$. Thus, the fluctuations we observe come from contributions from the inherent fluctuations and from the loss of energy due to Φ. The gravitational potential term wins, producing the negative sign and showing that photon overdense regions are actually cold spots on the CMB.

2 Putting the Integrated Sachs-Wolfe Effect (ISW) to Use

From the Integrated Sachs-Wolfe effect, the change in temperature due to the ISW in the $\hat{n}$is given by:

$$\Delta T_{ISW}(\hat{n}) = -2\int dD\dot{\Phi}(x(\hat{n}), D) \tag{4}$$

The gravitational potential is a function of position and also the lookback distance D. Further, in a flat, matter-dominated universe, the potential is constant in time, and so the ISW effect that is measured must necessarily originate at early times when radiation was significant, or at a late time when dark energy was greatest. The goal is to find the late ISW signal to make a measurement of dark energy.

Before that can happen, the primary anisotropy from the surface of last scattering must be removed. This is possible because the two anisotropies were imprinted on the CMB at different epochs in the universe, and therefore at different length scales, and so the primary anisotropy should be uncorrelated with general growth measurements of the CMB, including the effects from the ISW.

To carry out the correlation with the temperature map of the CMB, we first consider a measurement of gravitational lensing of the CMB by matter along the CMB photons' path to Earth. To model the lensing, the gradient of a scalar field ϕ, called the projected potential, is used. ϕ is related to the true gravitational potential as follows:

$$\phi(\hat{n}) = -2 \int dD \frac{D_{LSS} - D}{DD_{LSS}} \Phi(x(\hat{n}), D) \tag{5}$$

D_{LSS} is the distance to the surface of last scattering. From here, the projected potential map can be broken down into spherical harmonics, and then an angular power spectrum can be created, in a similar process as with the temperature map of the CMB itself. Having done this process, we have the gravitational lensing information needed to correlate with the temperature map. The expression for the power spectrum is a line-of-sight integral:

$$C_\ell^{\phi\phi} \sim \int dD D \Phi^2(k, D) (\frac{D_S - D}{DD_S})^2 P_\delta^2(k) \mid_{k=\ell \frac{H_0}{D}} \tag{6}$$

This expression is only true in the large ℓ limit, corresponding to a flat sky. Exact expressions are used when ℓ is small enough to matter. Finally, the cross correlation between the temperature and lensing has a similar flat sky form due to the ISW's effect on the temperature. That expression is the following:

$$C_\ell^{T\phi} \sim \int dD D \Phi(k, D) \dot{\Phi}(k, D) \frac{D_S - D}{DD_S} P_\delta^2(k) \mid_{k=\ell \frac{H_0}{D}} \tag{7}$$

Another observation to use is simply a count of galaxies on a portion of the sky. Fluctuations of the number density of galaxies should follow fluctuations in the gravitational potential on large enough scales. The auto-correlation power spectrum

$$C_\ell^{gg} \sim \int dD D \Phi^2(k, D) n_g^2(D) P_\delta^2(k) \mid_{k=\ell \frac{H_0}{D}} \tag{8}$$

This power spectrum depends on the potential, but modified by a window $n_g(D)$ which describes the observed distribution of galaxies. This spectrum is then correlated with the ISW temperature spectrum, and the lensing potential power spectrum, given by

$$C_\ell^{Tg} \sim \int dDD\Phi(k,D)\dot{\Phi}(k,D)n_g(D)P_\delta^2(k)\mid_{k=\ell\frac{H_0}{D}} \tag{9}$$

and

$$C_\ell^{g\phi} \sim \int dDD\Phi^2(k,D)n_g(D)\frac{D_S-D}{DD_S}P_\delta^2(k)\mid_{k=\ell\frac{H_0}{D}} \tag{10}$$

respectively.

A form that the dark energy equation of state parameter will take must also be chosen. Since there is little specific information about what it should look like, principle mode analysis can provide a way to choose some form that will be able to work with the experiment. Ultimately, a nice form is chosen as follows:

$$w(z) = w_0 + w_\alpha \frac{z}{1+z} \tag{11}$$

3 Results and Errors

Ultimately, constraint contours are constructed in the w_0-w_α plane simultaneously with constraints on the other cosmological parameters. CMB lensing and galaxy counts can realistically constrain w_0 to a precision of about ± 0.33. Additionally, without noise, over 40% of the improvement in this constraint comes from the $C_\ell^{T\phi}$ cross-correlation channel. However, there is some noise that is able to infiltrate the channel, and this will limit how well CMB lensing can constraint dark energy. Further, cross-correlating galaxies with the ISW effect was able to improve the constraint by over a factor of two by adding redshift information to the fits.

However, there are still some issues that merit attention. Galaxy surveys themselves can suffer from several problems which limit their effectiveness. Knowledge of the redshift distribution of the galaxies could pose a problem. Error in redshift can be converted to an error in angular scale of galaxy correlations, and these errors can then be compared to the instrument's shot noise. Fortunately in this case, for the angular scales in our survey, the shot noise is relatively high, so redshift errors of about one percent are acceptable. However, this situation will not always be the case, and future experiments could always reveal new problems in redshift calibration.

Additionally, all of the analysis in this work was under the assumption that there were no perturbations at all in the dark energy fluid. The cosmological constant model of dark energy has an equation of state value of $w = -1$ and is perfectly smooth. However, a cosmological constant is not the only proposed mechanism for dark energy. For example, scalar field quintessence models could provide the dark energy and can have density fluctuations. Not surprisingly, allowing dark energy to have its own fluctuations can significantly alter the results, especially during the late times when the structure formation driving the ISW is happening.

4 Conclusions

In principle, it is possible to set limits on the dark energy by using the technique of cross-correlation of the ISW effect with CMB lensing and galaxy counts. It should be possible to measure w_0 to ± 0.093 and w_α to ± 0.32. However, in practice the limits are about a factor of three worse, as we discovered. Dark energy's effect on the growth of structure, which isn't completely understood yet, has a significant effect on these measurements, and as a result the measurements also depend on the equation of state that is more complicated than the usual distance-redshift measurements. A reasonable next step is to take direct measurements of some of these structure growths, and hopefully begin to show how dark energy drives the acceleration of the expansion of the universe.

Despite these problems, there is room for improvement. One area is in the measurement of the lensing. The amount of lensing measured is very small, so small cosmic variance and resolution of the instruments become significant. This work used the CMB temperature map to reconstruct the lensing spectrum using four-point correlations. However, utilizing the three-point correlation function would possibly help extract information about lensing that is less noisy. Whether this can be done in practice remains an open question.

Additionally, due to the very large distance scales on which galaxy densities correlate with the integrated Sachs-Wolfe effect, the deviations from smooth background are very small, and so shot noise presents a noticable problem. A possible solution would be to gather more data about the galaxies than just their number density; good measurements of shapes and redshifts would be very useful in cutting down noise. However, making these sorts of observations would require dedicated telescopes and more resources than simply conducting a very large survey of galaxies, and so this is a difficult solution to undertake.

The Sachs-Wolfe effect presents a terrific opportunity to study many aspects of the cosmic microwave background. It is not only possible to study temperature fluctuations at the surface of last scattering, but also to study the evolution of structure and perturbations in the universe due to the integrated Sachs-Wolfe effect's continuous imprinting onto cosmic microwave background photons. In the future as a wider variety of observational apparatus and techniques become available, some of the present shortcomings will be addressed and the limits on the cosmological parameters, specifically on the state parameter of dark energy, can be made even more narrow. As time and technology progress, the Sachs-Wolfe effect will hopefully continue to refine cosmological parameters and teach us even more about the evolution of the universe.

5 References

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