Quantum 125c – Sean Carroll

4/3/17 ①

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Lecture One

Web page: http://preposterousuniverse.com/activities/physics125c

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Professor's office hours:
Fridays 10:30 – noon, Downs/Lauritsen 401

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Tuesdays 7:00 – 8:30pm, Downs/Lauritsen 4th floor

Grading: 70% Problem Sets, 30% Final Exam

Problem Sets: roughly weekly, handed out Wednesdays, due following Weds. @ 5:00 pm.

Usual 125 philosophy:

125 a/b take the non-relativistic Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\Psi(x,t) = i\hbar\,\partial_t \Psi(x,t)$$

and beat it to death.

Then 125c is a grab bag of special topics.

This course: <u>Subsystems & Entanglement.</u>

Rough Outline:

1. Basics of Subsystems & Entanglement
   → qubits, density matrices, entropy
2. Measurement
   → Decoherence, Bell's Theorem, etc.
3. Foundations
   → Everett, Bohm, Dynamical Collapse
4. Quantum Info
   → circuits, algorithms
5. Quantum Field Theory
   → free-field vacuum.

Why are subsystems a big deal?

1. Without subsystems, QM is trivial.

General Schrödinger equation:

$$\hat{H}|\Psi\rangle = i\partial_t|\Psi\rangle \qquad (\hbar = 1)$$

Linear in $|\Psi\rangle$! Easy to solve.

Energy eigen basis: $\hat{H}|E_a\rangle = E_a|E_a\rangle$

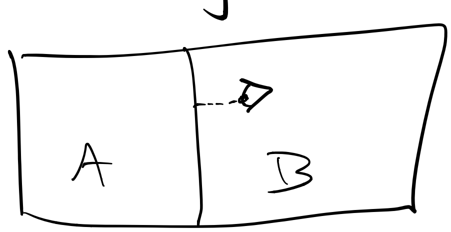
$$|\Psi(0)\rangle = \sum_a r_a e^{i\phi_a}|E_a\rangle$$

Then

$$|\Psi(t)\rangle = \sum_a r_a \underbrace{e^{-i(E_a t - \phi_a)}}_{\text{time evolution = boring phases}}|E_a\rangle$$

QM is interesting when one subsystem interacts with another. Then:

- Subsystem evolution isn't unitary.
- Interaction picks out a basis that might not be $\{|E_a\rangle\}$.



Why subsystems matter:

2. There is only one real "system" — the entire universe.

Classically: separate states for each subsystem. (Direct product structure.)

Hilbert space is a <u>tensor</u> product:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B .$$

If $\{|\phi_a\rangle_A\}$ is a basis for $\mathcal{H}_A$,
and $\{|\eta_b\rangle_B\}$ is a basis for $\mathcal{H}_B$,
elements of $\mathcal{H}$ are arbitrary superpositions:

$$|\Psi\rangle = \sum_{ab} \psi_{ab} \, |\phi_a\rangle_A \otimes |\eta_b\rangle_B .$$

Note! $\dim (A \times B) = \dim A + \dim B$.

$\dim (A \otimes B) = (\dim A) \cdot (\dim B)$. Bigger!

elements of $\mathcal{H}$ are typically <u>not</u> of the form

$$\left(\sum_a \psi_a^{(A)} |\phi_a\rangle_A \right) \cdot \left(\sum_b \psi_b^{(B)} |\eta_b\rangle_B \right) .$$

There is only one wave function — the wave function of the universe.

Why subsystems matter:

3 QM is mysterious because **measurement** is mysterious.

And measurement is all about interactions between subsystems.

$$\mathcal{H} = \mathcal{H}_{system} \otimes \mathcal{H}_{observer}$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_{sys} + |-\rangle_{sys}\right) \otimes |0_{ready}\rangle_{obs}$$

$$\xrightarrow{\text{time}} \frac{1}{\sqrt{2}}|+\rangle_{sys} \otimes |0_+\rangle_{obs} + \frac{1}{\sqrt{2}}|-\rangle_{sys} \otimes |0_-\rangle_{obs}$$

All the mystery of "interpreting" QM is how to think about this.

4 **Entanglement** opens new doors.

$\hookrightarrow$ a relation between subsystems.

Quantum teleportation, cryptography, computation — all about entanglement.

Maybe even the origin of spacetime itself.

"nearby" $\stackrel{?}{=}$ "more entangled"

Qubits

2-dim quantum systems. $\mathcal{H} = \mathbb{C}^2$.

[cf. 1 non-relativistic particle: $\dim \mathcal{H} = \infty$.]

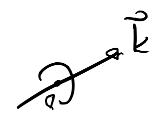
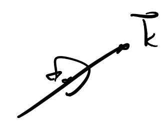
Classical bit: 0 or 1

Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, $|\alpha|^2 + |\beta|^2 = 1$

$\{|0\rangle, |1\rangle\}$ = "standard" or "computational" basis.

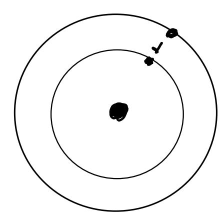
N.B.: # of distinct qubits $= \infty$.

Examples:

- photon polarization  or 

  $|0\rangle = |RH\rangle$ $\quad$ $|1\rangle = |LH\rangle$

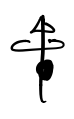
- atomic electron in lowest two energy orbitals

  $|E_0\rangle = |0\rangle$

  $|E_1\rangle = |1\rangle$

- Spin-½ fermions (electrons, ν's, quarks, nuclei, etc.)

   $|+z\rangle = |0\rangle$ $\quad$  $|-z\rangle = |1\rangle$

- Flux in a superconducting loop

  $|0\rangle = |\text{clockwise}\rangle$

  $|1\rangle = |\text{counterclockwise}\rangle$

Bloch Sphere

Global phase $e^{i\gamma}$ doesnt matter to $|\psi\rangle$. (7)

So choose $\alpha \in \mathbb{R}^+$ in $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$.

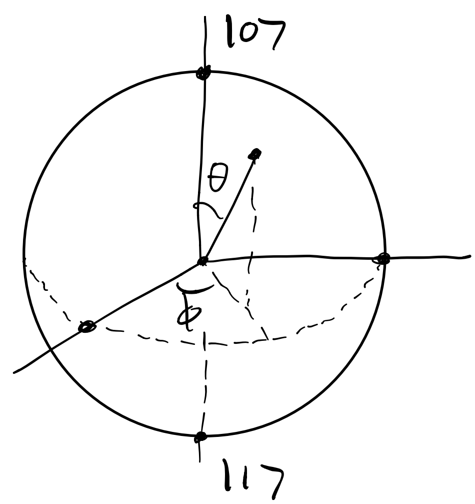
$\therefore$ we can write

$$\alpha = \cos\frac{\theta}{2}, \quad \beta = e^{i\phi}\sin\frac{\theta}{2},$$

$$0 \le \theta \le \pi, \quad 0 \le \phi \le 2\pi.$$

Parameterizes a sphere!

Bloch sphere:



$|0\rangle \leftrightarrow \theta = 0$, N. pole

$|1\rangle \leftrightarrow \theta = \pi$, S. pole

Qubits as spins in x/y/z

Pauli matrices:
$$X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$Y = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$Z = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Traceless, Hermitian, $\{\sigma_i, \sigma_j\} = 2\delta_{ij} \mathbb{1}_{2\times 2}$.

Eigenvectors:

X: $|{+x}\rangle \equiv |{+}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|{-x}\rangle \equiv |{-}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

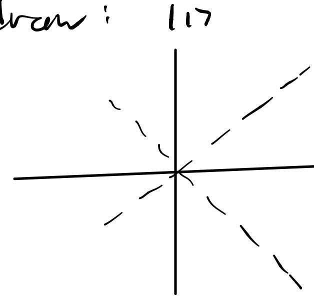
Y: $|{+y}\rangle \equiv |{+i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$

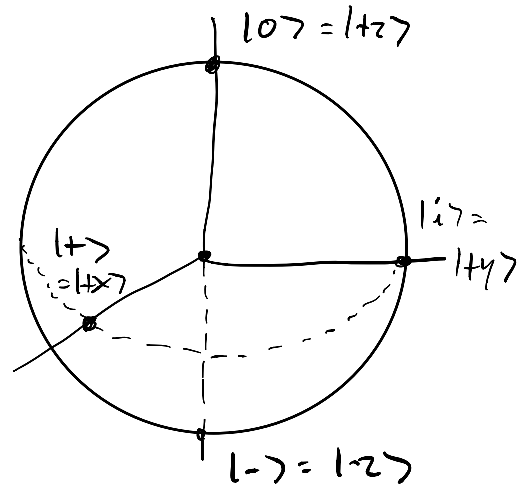
$|{-y}\rangle \equiv |{-i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$

Z: $|{+z}\rangle = |0\rangle$

$|{-z}\rangle = |1\rangle$

Sometimes we just draw:





Evolution

Schrödinger Eq. applies to qubits as it does to anything else.

$$\hat{H}|\psi\rangle = i\partial_t|\psi\rangle$$

Matrix form:

$$H = \begin{pmatrix} H_{00} & H_{01} \\ H_{10} & H_{11} \end{pmatrix}$$

Diagonalize: $\hat{H}|E_i\rangle = E_i|E_i\rangle$

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

$$|\psi(t)\rangle = \psi_1 e^{-iE_1 t}|E_1\rangle + \psi_2 e^{-iE_2 t}|E_2\rangle$$

But of course we can write time evolution in terms of a unitary operator:

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle = \hat{U}(t)|\psi(0)\rangle$$

$$\hat{U} = e^{-i\hat{H}t}, \quad \hat{U}^\dagger\hat{U} = \mathbb{1}$$

Often w/ qubits we'll just use "unitaries" directly, rather than worrying about $\hat{H}$.