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125c, Lecture Ten : 5/3/17

[References: Wallace, Schlosshauer]

Foundations / Interpretations of QM

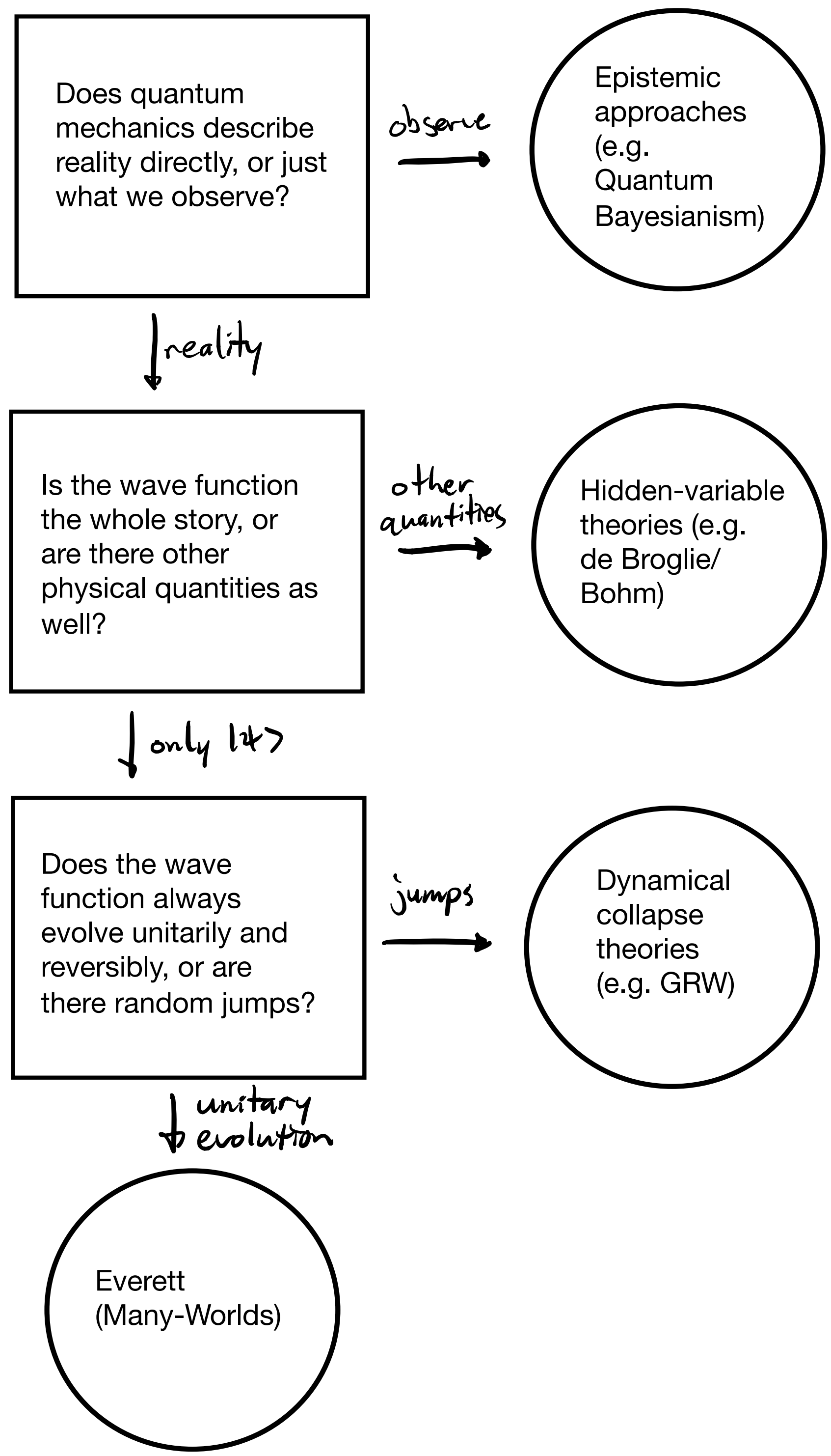
"Interpretations" can be misleading b/c it sounds like we're contemplating alternative ways of thinking about the <u>same</u> physical theory. For the most part, we'll actually be contrasting truly different theories.

A theory comes in three parts:

1. A set of observed phenomena to be explained. (Data.)
2. A formal mathematical structure. (Mathematical objects & equations.)
3. An "interpretation" connecting 1 to 2.

For most areas of science, 3 is obvious; for quantum mechanics, not so much.

Possibilities we will explore:



Everett (Relative-State, Many-Worlds)

A simple idea, at the level of formalism:

- The world is described by a wave function (quantum state) $|\Psi\rangle$, living in a Hilbert space $\mathcal{H}$.
- $|\Psi\rangle$ evolves according to the Schrödinger equation, $\hat{H}|\Psi\rangle = i\partial_t|\Psi\rangle$, for some Hamiltonian $\hat{H}$.

That's it!

Everything else – including measurement, "collapse," probabilities – is <u>derived</u> from this simple starting point.

Price we pay: the world we see is only a very tiny slice of the whole shebang.

Motivational Pep Talk

If you accept QM, you presumably accept that an electron can be in a superposition of spin-up and spin-down:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

If you further accept:

- Everything (macroscopic as well as microscopic) obeys the rules of quantum mechanics, and
- The wave fn. provides a complete, realist account of the world.

Then you should accept:

- A macroscopic apparatus can (in principle) be in a superposition of "having detected spin-up" & "having detected spin-down."

- A person can be in a superposition of "having seen the apparatus read spin-up" & "having seen the apparatus read "spin-down."
- The universe can be in a superposition of "containing a person who saw the apparatus read spin-up" and "containing a person who saw the apparatus read spin-down."

If you buy all that, the only question is whether such superpositions actually come into being through ordinary evolution of the wave function.

Our investigation of the measurement process implies that they clearly <u>do</u>.

Remember <u>decoherence</u>: $|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E$,

and

$$|\Psi\rangle = \left(\psi_1|\phi_1\rangle + \psi_2|\phi_2\rangle\right) \otimes |a_0\rangle \otimes |E_0\rangle$$

$$\rightsquigarrow \left(\psi_1|\phi_1\rangle|a_1\rangle + \psi_2|\phi_2\rangle|a_2\rangle\right) \otimes |E_0\rangle$$

$$\rightsquigarrow \psi_1|\phi_1\rangle|a_1\rangle|\ \ \rangle + \psi_2|\phi_2\rangle|a_2\rangle|E_2\rangle.$$

Decoherence results from the fact that the environment states are, in realistic circumstances, very close to orthogonal:

$$\langle E_1|E_2\rangle \approx 0. \quad \text{(extremely good approximation)}$$

The reduced density matrix $\hat{\rho}_{SA}$ is diagonal:

$$\hat{\rho}_{SA} = \begin{pmatrix} |\psi_1|^2 & 0 \\ 0 & |\psi_2|^2 \end{pmatrix}$$

<u>Standard interpretation</u>: this looks like a classical probabilistic ensemble, so let's pretend that it is, and that $|\psi_i|^2$ is the probability.

Everett: no, that's just a subsystem of a larger pure quantum state. The whole quantum state really exists:

$$|\Psi\rangle = \psi_1|\phi_1\rangle|a_1\rangle|E_1\rangle + \psi_2|\phi_2\rangle|a_2\rangle|E_2\rangle$$

An apparent problem immediately arises. Let's separate out a person, the observer, from the environment, so replace

$$\mathcal{H}_E = \mathcal{H}_p \otimes \mathcal{H}_e, \qquad |E_n\rangle = \underset{\text{observer}}{|p_n\rangle}\underset{\text{environment}}{|e_n\rangle}.$$

Then our post-decoherence state is

$$|\Psi\rangle = \psi_1|\phi_1\rangle|a_1\rangle|p_1\rangle|e_1\rangle + \psi_2|\phi_2\rangle|a_2\rangle|p_2\rangle|e_2\rangle.$$

So the person is represented in two different ways, $|p_1\rangle$ & $|p_2\rangle$, in the two terms of this superposition.

But we don't ever <u>feel</u> like we're in a superposition. We always perceive definite measurement outcomes!

Everett's (ingenious) resolution: a "person" is not a state in $\mathcal{H}_P$ (or even a reduced density matrix $\hat{\rho}_P = \mathrm{Tr}_{SAe} |\Psi\rangle\langle\Psi|$).

Rather, the single person that started in

$$|\Psi(0)\rangle = (\psi_1|\phi_1\rangle + \psi_2|\phi_2\rangle)\,|a_0\rangle \underbrace{|p_0\rangle}_{\text{person}} |e_0\rangle$$

evolves to <u>two</u> people — one for each term in the superposition:

$$|\Psi(T)\rangle = \psi_1|\phi_1\rangle|a_1\rangle\underbrace{|p_1\rangle}_{\text{person }1}|e_1\rangle + \psi_2|\phi_2\rangle|a_2\rangle\underbrace{|p_2\rangle}_{\text{person }2}|e_2\rangle.$$

More generally: at decoherence, the wave function <u>branches</u> into orthogonal components. Each branch has an existence of its own, and should be thought of as a separate <u>world</u>.

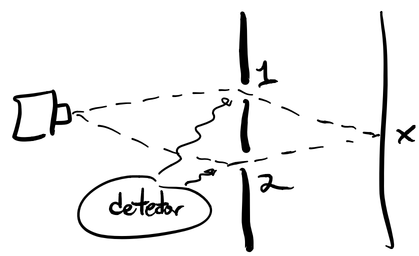
Crucially important: branches (worlds) are <u>not put in by hand</u>. They arise naturally through unitary evolution.

<u>Why</u> is it okay to consider decohered branches as separate worlds?

→ Because they don't interfere with (or in other ways influence) each other.

Remember the double-slit experiment.

If detector doesn't observe which slit the particle goes through → interference.



If the detector does observe which slit — i.e., becomes entangled with the particle, then decoheres via entanglement with the environment — interference disappears.

Evolution of any one branch is (to an excellent approximation) independent of the existence of other branches.

Thus branches — components of the non-environment density matrix that have decohered — are like separate worlds.
[Though Everett himself didn't invoke decoherence.]

So Everettian quantum mechanics (EQM) <u>posits</u>:

① $|\Psi\rangle \in \mathcal{H}$

② $\hat{H}|\Psi\rangle = i\partial_t|\Psi\rangle$

Then we <u>derive</u> (or try to):

③ $\mathcal{H} = \underbrace{\mathcal{H}_S}_{\text{"system" = macroscopic reality}} \otimes \underbrace{\mathcal{H}_E}_{\text{"environment"}}$

④ Branching via decoherence:

$$|\Psi\rangle = (\psi_1|\phi_1\rangle + \psi_2|\phi_2\rangle)|e_0\rangle \rightsquigarrow \psi_1|\phi_1\rangle|e_1\rangle + \psi_2|\phi_2\rangle|e_2\rangle$$

$$\therefore \hat{\rho}_S = \begin{pmatrix} |\psi_1|^2 & 0 \\ 0 & |\psi_2|^2 \end{pmatrix}$$

⑤ Observers branch along with everything else, and each post-branching observer witnesses a definite measurement outcome.

This is a lot to swallow, and we should think over possible objections. There are bad objections to EQM, and some legit worries.

Bad Objections to EQM

1. Ontological Extravagance — postulating such a huge number of worlds goes against Occom's Razor.

   Response: but you <u>don't</u> postulate that. You postulate Hilbert space & Schrödinger's equation; the worlds come for free. The theory itself is extremely simple.

2. It's untestable.

   Response: of course it's testable. Again, the entire theory is (1) $|\psi\rangle \in q_d$ and (2) $\hat{H}|\psi\rangle = i\partial_t|\psi\rangle$. Both of those are very testable! The double-slit experiment is a great test of the principles of EQM.

"Ah," but you say, "you can't test the idea that the other branches are really there." Three answers:

1) A theory isn't bad if some of it's predictions aren't testable. It's bad if its basic principles have no empirical implications. EQM's obviously do.

2) Other branches are detectable, in principle if not in practice. Decoherence is usually not perfect. Interference between branches is conceivable, it's just exponentially tiny.

3) If you think the other branches aren't there, tell me what happened to them. Invent a disappearing-worlds theory. (Which people have done.)

Many people just don't like EQM because the idea of other copies of themselves is creepy. Science doesn't care.

Good questions for EQM.

① Branching, ② Structure, ③ Probability.

① Branching: When exactly does the wave function branch? How many branches are there? Is branching non-local?

Subtle questions that even committed Everettians disagree about, except that they don't matter too much.

- When? When a system becomes entangled with an environment and decoheres. (Remember, lots of quantum evolution does not increase entanglement.) Of course these are approximate notions – but that's fine. We should be comfortable with approximations (e.g. stat mech), and these are very good ones.

- How many branches are there?

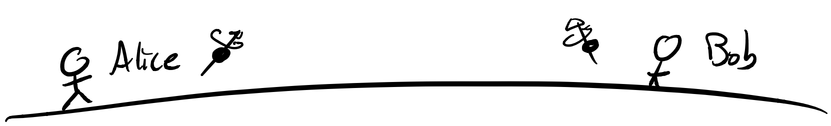
  This has no clear answer in an infinite-dim. Hilbert space. But we can still say "The |amplitude|^2 for branches with outcome A is 0.45 the |amplitude|^2 for those with outcome B," which is all that matters. If dim $\mathcal{H}$ = finite, maybe you can be more definite.

- Is branching nonlocal?

  I think so – but others disagree.

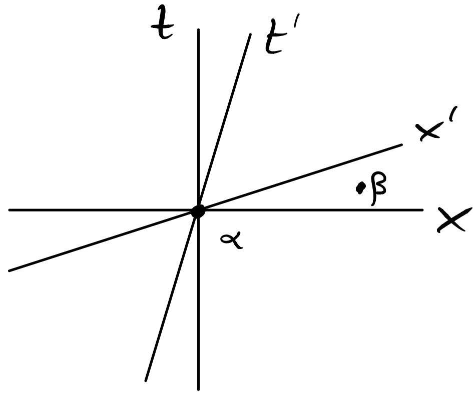
  This is already familiar from EPR:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B\right)$$



If Alice measures, Bob is "instantly" in one state or the other. That corresponds to Alice's branching instantly affecting the wave function everywhere.

How can we talk about "instantaneous" in a world with special relativity?

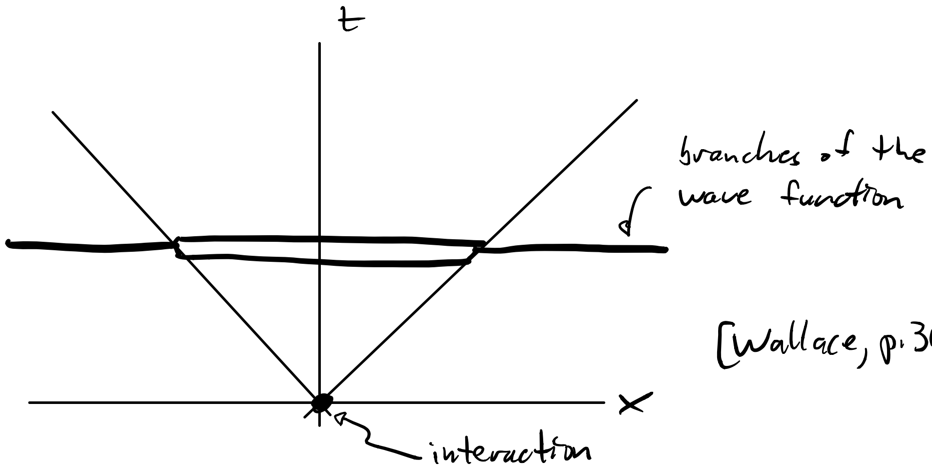


α happens before β according to t, but after β according to t'.

Physics can be Lorentz-invariant without being <u>manifestly</u> Lorentz-invariant. I.e. Lorentz transformations can lead to distinct and apparently different descriptions of what happens, but imply the same <u>observable</u> effects. (Bob cannot observe when Alice branches the wave function.)

The Schrödinger eq. $\hat{H}|\psi(x,t)\rangle = i\partial_t|\psi(x,t)\rangle$ treats x & t very differently, but if $\hat{H}$ is the Hamiltonian of a Lorentz-invariant theory, observables will be Lorentz-invariant.

A possible alternative view:
branching is local, and spreads out inside light cones, emanating from whatever interaction caused decoherence.



[Wallace, p. 307.]

I think this is too tied to the structure of spacetime, which should itself ultimately be emergent (in quantum gravity). But worth thinking about.

Next time: questions about structure & probability.