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125c, Lecture Twelve: 5/10/17.

Finishing up probability in MWI.

- Self-locating uncertainty: even if you know $|\Psi\rangle$ exactly, there's something you might not know: which branch you are on. How do we assign credences in such a case?

- ESP (Epistemic Separability Principle): whatever credences you assign, what goes on elsewhere shouldn't matter.

We looked at an example with equal amplitudes for two branches:

$$|\Psi_A\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle|x_1\rangle|e_1\rangle|p_0\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle|x_2\rangle|e_2\rangle|p_0\rangle$$

$$|\Psi_B\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle|x_2\rangle|e_1\rangle|p_0\rangle + \frac{1}{\sqrt{2}}|\phi_2\rangle|x_1\rangle|e_2\rangle|p_0\rangle.$$

By first treating $|x_i\rangle$ as part of the environment, then $|\phi_i\rangle$, we argued that

$$p(\phi_1|A) = p(\phi_2|A) = \frac{1}{2} = (1/\sqrt{2})^2.$$

This is of course just what we expect from the Born Rule - probability is $|\text{amplitude}|^2$.

Unequal amplitudes? Trick doesn't work, b/c (for example) they're would be no reason to have $p(x_2|A) = p(x_2|B)$. (Tracing over $|\phi\rangle$'s yields different density matrices.)

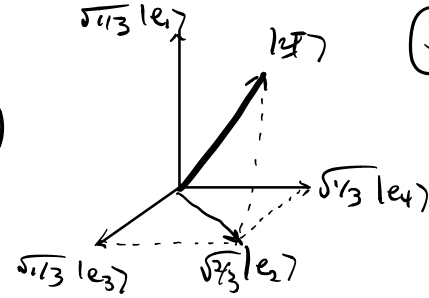
But another trick comes along: decompose components into sub-components until all amplitudes are equal.

E.g.

$$|\Psi\rangle = \sqrt{\tfrac{1}{3}}\,|\phi_1\rangle|x_1\rangle|e_1\rangle + \sqrt{\tfrac{2}{3}}\,|\phi_2\rangle|x_2\rangle|e_2\rangle$$

We can always write

$$|e_2\rangle = \frac{1}{\sqrt{2}}\left(|e_3\rangle + |e_4\rangle\right)$$



with $\langle e_3|e_4\rangle = \langle e_3|e_1\rangle = \langle e_4|e_1\rangle = 0$.

$$|\Psi\rangle = \sqrt{\tfrac{1}{3}}\,|\phi_1\rangle|\chi_1\rangle|e_1\rangle + \sqrt{\tfrac{1}{3}}\,|\phi_2\rangle|\chi_2\rangle|e_3\rangle + \sqrt{\tfrac{1}{3}}\,|\phi_2\rangle|\chi_2\rangle|e_4\rangle$$

Now we can show that each component should get equal probability: $p(\phi_2) = 2\,p(\phi_1)$. Multiplying amplitude by $\sqrt{2}$ multiplies probability by 2. The Born Rule!

<u>Worries</u>:

- what if $|\alpha|^2/|\beta|^2$ is irrational?
  - Rationals are dense in $\mathbb{R}$, that should be enough.
- Are there really two copies of $|p_0\rangle$?
  - I think so, though YMMV.
- Can't I put some <u>other</u> measure on my credences?
  - Maybe. But this is unique given ESP, fits the structure of QM, and also fits the data. Why not declare victory?

Credences Strategy #2: Decision Theory.

Pioneered by David Deutsch; improved by David Wallace. We'll discuss the earliest/simplest / most primitive approach.

Decision theory: life is a game. To every action α, there is a set of possible outcomes $\{\sigma_i\}$, each of which has a probability p_i of occuring, and a utility (value, payoff) V_i if it does. The expected value EV of an action is

$$EV(\alpha) = \sum_i p_i V_i \; .$$

The rules of decision theory say that rational agents prefer actions with higher expected value, and are indifferent between actions of equal value.

Note "value" $\neq$ "money", since the value of money is non-linear: \$2 million isn't twice as valuable as \$1 million (marginal utility).

Consider EQM, and take for granted the usual story about measurements corresponding to decoherence & branching.

Let's imagine we can construct a "value operator" $\hat{X}$, s.t. eigenstates of $\hat{X}$ have expected value equal to the eigenvalue:

$$\hat{X}|x_1\rangle = x_1|x_1\rangle \longrightarrow EV[|x_1\rangle] = x_1 .$$

What we want to show is that, in superpositions of such eigenvectors, we should attach a value that is related to the amplitudes just as probabilities are in the Born Rule:

$$EV[\alpha|x_1\rangle + \beta|x_2\rangle] = |\alpha|^2 x_1 + |\beta|^2 x_2$$

Comparing with the general EV formula above, we see that it is rational to act <u>as if</u> $p_1 = |\alpha|^2$, $p_2 = |\beta|^2$.

First note: given an action with a certain set of possible payoffs, the value of an action with <u>minus</u> those payoffs is just minus the value of the original.

In our example, take $\alpha = \beta = 1/\sqrt{2}$:

$$EV\left[\tfrac{1}{\sqrt{2}}\left(|{-x_1}\rangle + |{-x_2}\rangle\right)\right] = -EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)\right]. \circledast$$

Another principle of decision theory: if we increase the payoff of every outcome by a fixed amount k, that's equivalent to the original action plus receiving value k.

$$EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1+k\rangle + |x_2+k\rangle\right)\right] = EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)\right] + k.$$

Now just set $k = -x_1 - x_2$:

$$EV\left[\tfrac{1}{\sqrt{2}}\left(|{-x_2}\rangle + |{-x_1}\rangle\right)\right] = EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)\right] - x_1 - x_2.$$

Plug into ⊛ to obtain

$$-2\,EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)\right] = -x_1 - x_2$$

$$\Rightarrow EV\left[\tfrac{1}{\sqrt{2}}\left(|x_1\rangle + |x_2\rangle\right)\right] = \tfrac{1}{2}(x_1 + x_2).$$

That's just the Born Rule.

Extensions to unequal amplitudes can be constructed as before.

Upshot: While the dynamics in MWI are deterministic, situations arise where you don't know everything & need to reason under conditions of uncertainty. Good arguments suggest that the right strategy for assigning credences is the Born Rule. (Whether those arguments are good enough is, of course, controversial.)

→ It can't be denied that this notion of probability is different than the standard idea of truly stochastic dynamics.

Alternatives to MWI

We'll briefly consider three approaches:

1. de Broglie/Bohm (hidden variables)
2. Ghirardi-Rimini-Weber (dynamical collapse)
3. QBism (epistemic).

So let's start with:

"Hidden-Variable" Theories, in particular

de Broglie-Bohm or simply "Bohmian mechanics."

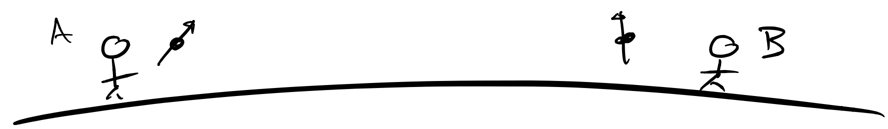
Note this is the historical name, but many proponents don't like "hidden variables," in part because these variables are the ones we actually observe. So we'll follow more modern usage & refer to "Bohmian mechanics."

[Workers in the field speak of taking existing QM models and "Bohm-izing" them.]

First an important preamble:

Bell's Theorem.

Remember EPR: Alice, Bob, & their entangled qubits.



EPR themselves actually dealt with position / momentum entanglement. The (much more straightforward) case of qubits was thoroughly studied by John Bell, so much so that maximally entangled qubits are called "Bell pairs" and the states

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle \right),$$

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle \right)$$

are called "Bell states." (David Bohm also studied this setup.)

Here's what EPR themselves said:

1. If, given a system in some state, we can predict with certainty the outcome of a measurement, that outcome corresponds to an "element of physical reality."

   E.g. for one qubit in $|\psi\rangle = |0\rangle = |+z\rangle$, "the spin in the z-direction" is an element of physical reality, "spin in the x-direction" is not (as far as we know).

2. If a theory is "complete," every element of physical reality must have a counterpart in the theory.

The $|\psi\rangle = |+z\rangle$ example shows that, if QM is complete, then commuting operators correspond to elements of physical reality, but non-commuting ones need not.

Think of Alice & Bob in the state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Imagine Alice measures $\hat{\sigma}_z^{(A)}$ on her qubit, and obtains the result $+1$.
Then we know that if Bob measures $\hat{\sigma}_z$, he will certainly measure $+1$ also.
(Not necessarily spooky-action-at-a-distance, since we're not assuming all the formalism of ordinary QM, just the experimental predictions. I.e. maybe there are hidden variables.) So according to EPR, when Alice finds $\sigma_z^{(A)} = \pm 1$, there is an element of reality corresponding to $\sigma^{(B)} = \pm 1$.
(Some element of reality @ B either way.)

But what if Alice measures $\hat{\sigma}_x^{(A)}$?

Basis change:

$$|+\rangle = |+x\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$|-\rangle = |-x\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Change bases for Alice & Bob, and plug into Bell state, obtaining

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|++\rangle + |--\rangle\right)$$

(still entangled).

So now imagine that Alice measures $\hat{\sigma}_x^{(A)}$. Again, she gets some definite result. Therefore we know what Bob will get if he measures $\hat{\sigma}_x^{(B)}$, with certainty. So there is some element of reality corresponding to Bob's $\hat{\sigma}_x^{(B)}$.

But – tricky part here — Bob could be very far from Alice. In EPR's eyes, then, Alice's measurements didn't affect the state of Bob's qubit. Therefore, those elements of reality corresponding to $\hat{\sigma}_z^{(B)}$ and $\hat{\sigma}_x^{(B)}$ had to have been there all along (even if we didn't know).

Therefore: quantum mechanics is incomplete.

Einstein suspected, but never fully developed, that some kind of hidden-variable trickery was going on.

In 1964, Bell turned EPR around: no theory with local hidden variables (the kind Einstein would presumably have preferred) can exactly reproduce quantum mechanics. That's Bell's Theorem.

Original version: give Alice & Bob a Bell pair, and imagine they measure the spin along different axes, tilted with respect to each other by an angle θ. For $\theta = 0°$ or $180°$, their answers are exactly correlated; for $90°$ or $270°$, completely uncorrelated. Near $0°$ or $180°$, Bell proved that they are more correlated in QM than they possibly could be in any theory of local hidden variables.

The result is a set of "Bell inequalities," that the correlations have to be less than certain values in local hidden-variable theories. QM violates the Bell inequalities, and this was experimentally confirmed finally in 2015.

Simpler, non-probabilistic version of Bell's result (Greenberger-Horne-Zeilinger, as modified by Mermin). Consider three qubits, A, B, C, in a GHZ state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right).$$

If Alice measures $\hat{\sigma}_z^{(A)}$, we know what Bob & Charley will get for $\hat{\sigma}_z^{(B)}$ & $\hat{\sigma}_z^{(C)}$.

Rewrite in the $|\pm x\rangle = |\pm\rangle$ basis:

$$|\psi\rangle = \frac{1}{2}\left(|++-\rangle + |+-+\rangle + |-++\rangle + |---\rangle\right).$$

Notice that the product of all three spins is always -1:

$$\sigma_x^{(A)}\sigma_x^{(B)}\sigma_x^{(C)} = -1.$$

We can play a similar game where one of A/B/C measures $\hat{\sigma}_x$, and the other two measure $\hat{\sigma}_y$. Tedious math, but we end up with the following relations in the GHZ state:

$$\sigma_x^{(A)} \sigma_x^{(B)} \sigma_x^{(C)} = -1.$$

$$\sigma_x^{(A)} \sigma_y^{(B)} \sigma_y^{(C)} = +1.$$

$$\sigma_y^{(A)} \sigma_x^{(B)} \sigma_y^{(C)} = +1.$$

$$\sigma_y^{(A)} \sigma_y^{(B)} \sigma_x^{(C)} = +1.$$

EPR wanted to believe that these products, since they are always true, correspond to elements of reality. But multiply the left-hand sides together:

$$\left(\sigma_x^{(A)}\right)^2 \left(\sigma_x^{(B)}\right)^2 \left(\sigma_x^{(C)}\right)^2 \left(\sigma_y^{(A)}\right)^2 \left(\sigma_y^{(B)}\right)^2 \left(\sigma_y^{(C)}\right)^2$$

This $= +1$, as the product of squares of ± 1. But the product of the right-hand side $= -1$. So they can't <u>simultaneously</u> be true.