Sean Carroll, seancarroll@gmail.com

125c, Lecture Thirteen: 5/15/17

Alternatives to MWI.

Everettian QM is simple to formulate ($|\psi\rangle \in \mathcal{H}$; $\hat{H}|\psi\rangle = i\partial_t|\psi\rangle$) but, for some people, hard to swallow. It's worth contemplating alternatives.

Problem: if you believe $|\psi\rangle$ represents reality, and obeys the Schrödinger equation, multiple worlds seem pretty automatic.

Strategies:

1. Add variables that pick out a single world (Bohmian mechanics).
2. Change Schrödinger equation (dynamical collapse / GRW).
3. Deny $|\psi\rangle$ represents reality (epistemic QM / QBism).

Bohmian Mechanics (hidden variables)

Also "de Broglie/ Bohm." Einstein arguably pioneered the idea; de Broglie invented a version, then abandoned it; Bohm rediscovered in 1952; championed later by Bell.

One caveat: our target here is the QM of non-relativistic spinless particles. Bohm-izing a theory is a nontrivial task, since you have to think carefully about what the new variables should be.

Remember that Bell's theorem implies hidden variables can't be local. The Bohmian strategy is that variables can be individual particle positions, but they interact non-locally.

So consider N particles, with positions

$$\{\vec{Q}_j\} = \{\vec{Q}_1, \dots \vec{Q}_N\} \in \mathbb{R}^{3N}$$

There is also an N-particle wave function,

$$\Psi(\vec{x}_1, \vec{x}_2, \dots \vec{x}_N) : \mathbb{R}^{3N} \to \mathbb{C}.$$

Note that the $\{\vec{x}_j\}$ specify any point in configuration space, while the notation $\{\vec{Q}_j\}$ means the actual positions of the particles.

Two equations. Schrödinger's equation as usual:

$$\hat{H}\Psi = i\partial_t \Psi,$$

where

$$\hat{H} = -\sum_{j=1}^{N} \frac{1}{2m_j} \vec{\nabla}_j^2 + V(\vec{x}_1, \dots \vec{x}_N),$$

where $\vec{\nabla}_j^2 \equiv \frac{\partial^2}{\partial x_j^2} + \frac{\partial^2}{\partial y_j^2} + \frac{\partial^2}{\partial z_j^2}$.

There is also the guidance equation, or "Bohm's Law of Motion":

$$\frac{d\vec{Q}_j}{dt} = \frac{1}{m_j}\,\mathrm{Im}\left(\frac{\vec{\nabla}_j \Psi}{\Psi}\right)(\{\vec{Q}_1 \cdots \vec{Q}_N\}(t))$$

Where did this ugly equation come from? Different routes, here is one heuristic idea. We want an equation for $d\vec{Q}_j/dt$ that depends on $\Psi(\{\vec{x}_j\}, t)$.

- Gradient $\vec{\nabla}_j$ is natural as a "push," and more formally to match properties under rotations. (I.e. both sides are vectors.)
- Ψ in denominator because magnitude of Ψ shouldn't matter.
- Imaginary part because of time reversal: $t \to -t$, $\Psi \to \Psi^*$.

Note:

- Equations are perfectly deterministic, for both Ψ and $\{\vec{Q}_j\}$.
- Particles are guided by Ψ, but don't affect it — the $\{\vec{Q}_j\}$ don't appear in Schrödinger's equation. Therefore sometimes called "pilot-wave theory."
- Guidance equation says that the velocity $\dot{Q}_j$ is affected, via the wave function, by all the other $\vec{Q}_{k\neq j}$. That's the non-locality.
- "Wave/particle duality" becomes trivial — there are particles, guided by a wave!

How does Bohm recover conventional QM?

Born Rule is easy (if somewhat ad hoc).

We measure the positions $\{\vec{Q}_j\}$, but don't know what they start out as.

But there is an "equivariance" feature:

→ If the initial configuration $\{\vec{Q}_j(t_0)\}$ is chosen randomly with respect to the probability distribution $|\Psi(\{\vec{x}_j\}, t_0)|^2$,

then later configurations $\{\vec{Q}_j(t)\}$ are random with respect to $|\Psi(\{\vec{x}_j\}, t)|^2$.

So if we start with a config that yields Born-Rule statistics we maintain that feature.

We don't have to start with such an "equilibrium" configuration, but if we do, Bohm is empirically indistinguishable from textbook QM.

What about "wave function collapse?" (7)

Divide the universe into "system" $\{\vec{X}_\alpha\}$

and "apparatus" $\{\vec{Y}_n\}$, so $\{\vec{Q}_j\} = \{\vec{X}_\alpha, \vec{Y}_n\}$.

The story is much like von Neumann measurement. We start with an unentangled wave function:

$$\Psi(t=0) = \left[\sum_\alpha \psi_\alpha(\{\vec{x}_\alpha\})\right] \cdot \phi_0(\{\vec{y}_n\})$$

Unitary evolution:

$$\Psi(t_1) = \sum_\alpha \left[\psi_\alpha(\{\vec{x}_\alpha\})\, \phi_\alpha(\{\vec{y}_n\})\right]$$

possible pointer readouts.

But now we read the pointer, measuring $\{\vec{y}_n\} = \{\vec{Y}_n\}$. If it's a good pointer, the functions ϕ_α will be non-overlapping for different $\{\vec{y}_n\}$'s, so the specific values $\{\vec{Y}_n\}$ pick out a "branch" ϕ_{α_0}.

We can define the conditional wave fn. (8)
for $\{\vec{x}_a\}$, given $\{\vec{Y}_n\}$, as

$$\psi(\{\vec{x}_a\}) = \Psi(\{\vec{x}_a\}, \{\vec{Y}_n\}).$$

If $\{\vec{Y}_n\}$ picks out ϕ_{α_0}, this is simply

$$\psi(\{\vec{x}_a\}) = \psi_{\alpha_0}(\{\vec{x}_a\})\,\phi_{\alpha_0}(\{\vec{Y}_n\}).$$

After we've measured the $\{\vec{Y}_n\}$'s, this conditional wave function can be used to describe the motion of the $\{\vec{X}_a\}$'s, since the rest of the wave function is orthogonal. It's as if the wave function has collapsed, though it's really deterministic evolution plus orthogonality (similar to the decoherence story).

Dynamical Collapse Models:

(GRW: Ghirardi, Rimini, Weber 1986.)

Back to just a state vector $|\Psi\rangle$, but now two types of evolution:

1. Usually the system evolves smoothly under the Schrödinger Equation.
2. At rare, random intervals, the system undergoes a "hit," spontaneously collapsing the wave function.

What is a "hit"? Consider an N-particle wave function $\Psi(\vec{x}_1, \vec{x}_2 \cdots \vec{x}_N)$. Choose one particle i, and a spatial location $\vec{q}$ chosen from a probability distribution

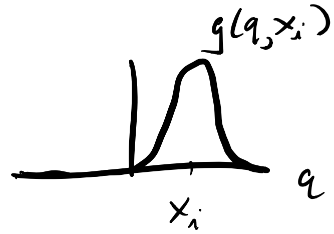
$$P(\vec{q}) = \int d^3x_1\, d^3x_2 \cdots [\cancel{d^3x_i}] \cdots d^3x_N \left|\Psi(\vec{x}_1, \cdots, \vec{x}_i = \vec{q}, \cdots \vec{x}_N)\right|^2.$$

Then a hit sends

$$\Psi(\vec{x}_1 \cdots \vec{x}_N) \to g(\vec{q}, \vec{x}_i)\, \Psi(\vec{x}_1 \ldots \vec{x}_n),$$

where g is localized near $\vec{x}_i = \vec{q}$:

$$g(\vec{q}, \vec{x}_i) = K e^{-(\vec{q}-\vec{x}_i)^2/2d^2},$$

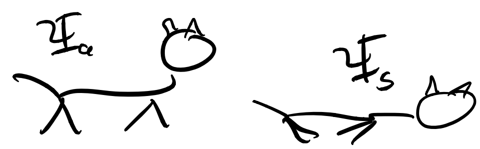


with K a normalization constant and $d \sim 10^{-5}$ cm the localization accuracy.

Hits are rare per particle: $\sim 10^{-16}\, s^{-1}$, or ~ 100 times in the age of the universe. But $\sim 10^{8}$ hits happen per second in a macroscopic system with 10^{24} particles.

Macroscopic objects are held together by forces. I.e. inter-particle distances are relatively well-defined. If we consider an object (a rock, Schrödinger's cat) in a superposition of different spatial locations, the location of each particle is highly entangled with the others.

$$\Psi_{cat}(\vec{x}_1, \dots \vec{x}_N) = \frac{1}{\sqrt{2}}\Psi_a(\vec{x}_1 \dots \vec{x}_N) + \frac{1}{\sqrt{2}}\Psi_s(\vec{x}_1 \dots \vec{x}_N)$$



Localizing just <u>one</u> particle collapses the entire cat wave function. Not pretty, but it works.

GRW is unambiguously different from standard QM. Experiments are ongoing to rule it out (or in).

QBism (formerly Quantum Bayesianism)

→ an epistemic approach.

(Note: QBism isn't the only epistemic approach, just a popular one.)

An utterly different point of view.

In QBism, $|\Psi\rangle$ doesn't represent reality. Rather, it encapsulates our knowledge of reality. It's a tool for calculating the probabilities of measurement outcomes. There is not "a wave function of the universe"; different observers may even use different wave functions.

- "Reality differs from one agent to another."
- QM is "a personal mode of thought."

A big benefit: "wave function collapse" makes much more sense.
If $|\psi\rangle$ encapsulates our <u>knowledge</u>, then of course it changes suddenly when we learn something new.
No "spooky action at a distance": we just know more about the world.

Fundamental Ingredients of QBism:

1. A set of <u>agents</u>, who have
2. <u>Beliefs</u> ($|\psi\rangle$, or really $\hat{\rho}$), and accumulate
3. <u>Experiences</u> (measurement outcomes).

In QBism, QM is just a way for agents to organize their beliefs and update them in light of new experiences.

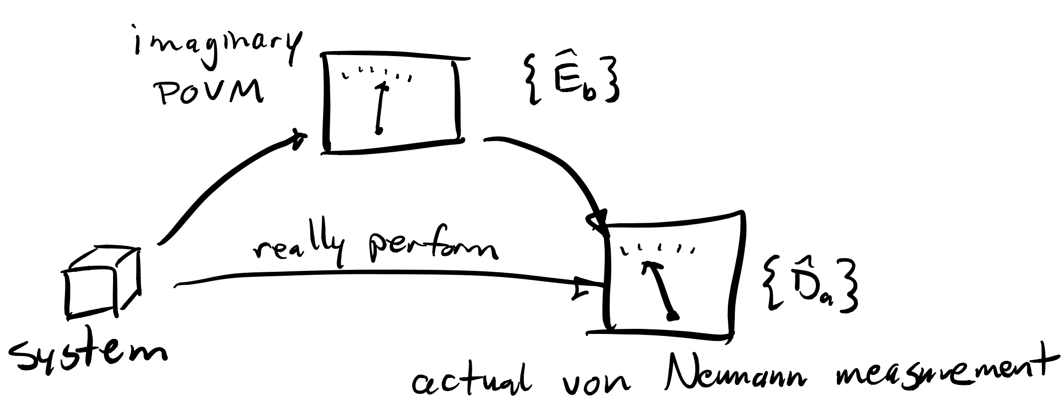
Imagine our agent wants to do a standard von Neumann PVM measurement, with possible outcomes $\{\hat{D}_a\}$, $a = 1 \cdots d$, $d = \dim \mathcal{H}_{\text{system}}$.

We want to ask about the probabilities $p(D_a)$, and hope they can be written as

$$p(D_a) = \mathrm{Tr}(\hat{D}_a \hat{\rho}) \quad \text{[the Born Rule]}$$

for some density operator $\hat{\rho}$.

But QBism asks: what if we <u>imagined</u> (but didn't actually perform) an intermediate measurement $\{\hat{E}_b\}$?



If we had done that measurement, obtaining outcomes $\{E_b\}$ with probabilities $q(E_b)$, we could decompose the probability for the subsequent $\{D_a\}$ measurement as

$$q(D_a) = \sum_b q(E_b)\, q(D_a | E_b).$$

In QM we don't expect these processes to give the same answer — the real process could feature interference! — so in general

$$\underset{\text{(actual)}}{p(D_a)} \neq \underset{\text{(hypothetical)}}{q(D_a)}.$$

QBism shows something interesting. Consider a particular kind of POVM for the counterfactual measurement $\{\hat{E}_b\}$ – a SIC ("seek").

or "Symmetric Informationally-Complete"

POVM. For $\dim \mathcal{H}_{\text{system}} = d$, that's a set of d^2 non-orthogonal operators

$$\hat{E}_b = \frac{1}{d} |\psi_b\rangle\langle\psi_b|,$$

with

$$|\langle\psi_b|\psi_c\rangle|^2 = \begin{cases} 1, & b=c \\ \frac{1}{d+1}, & b \neq c. \end{cases}$$

Check: $\sum_b \hat{E}_b = \mathbb{1}_d$.

Then two nice things:

1) Original density operator can be expressed in terms of hypothetical probabilities $q(E_b)$:

$$\hat{\rho} = \sum_{b=1}^{d^2} \left[d(d+1)\, q(E_b) - 1 \right] \hat{E}_b .$$

2) Actual probabilities can also be expressed in terms of hypotheticals:

$$p(D_a) = (d+1)\, q(D_a)$$

$$= (d+1) \sum_{b=1}^{d^2} q(D_b)\, q(E_a|D_b) - 1 .$$

The left-hand side here is the ordinary Born-Rule probability; the RHS is a sum over conditional probabilities for a measurement we didn't perform.

This lends support to the idea that QM should simply be thought of as an interesting addition to classical probability theory.

Also possible: QBism has merit as a useful formalism describing a real world that follows the rules of Everett or Bohm.