Sean Carroll, seancarroll@gmail.com

125c, Lecture Nine : 5/1/17

Finish up <u>decoherence</u>. [Schlosshauer is a good reference here.]

Decoherence = loss of purity in a subsystem due to entanglement with an environment.

Essential to connecting the behavior of quantum states to the approximately-classical behavior of the macroscopic world.

Think of the operator-sum representation (OSR) of a quantum channel:

$$\mathcal{E}(\hat{\rho}) = \sum_a \hat{K}_a \hat{\rho} \hat{K}_a^+ .$$

If there's only one Kraus operator $\hat{K}$, then $\mathcal{E}(\hat{\rho}) = \hat{K}\hat{\rho}\hat{K}^+$, and $\sum_a \hat{K}_a^+ \hat{K}_a = \hat{K}^+\hat{K} = \mathbb{1}$ implies $\hat{K}$ is <u>unitary</u>.

So <u>one Kraus operator</u> $\rightarrow$ unitary evolution $\rightarrow$ no decoherence.

Multiple $\hat{K}_a$'s are needed for decoherence.

Photons bouncing off rocks:

If rock's wave function is delocalized, photons interact in ways that depend on its position.

$$|0\rangle_R \sim \int d^3x\, e^{-(\vec{x}-\vec{x}_1)^2}|x\rangle, \quad |1\rangle_R \sim \int d^3x\, e^{-(\vec{x}-\vec{x}_2)^2}|x\rangle$$

Dephasing provides a good model:

$$|0\rangle_R|0\rangle_E \to \sqrt{1-p}\,|0\rangle_R|0\rangle_E + \sqrt{p}\,|0\rangle_R|1\rangle_E$$

$$|1\rangle_R|0\rangle_E \to \sqrt{1-p}\,|1\rangle_R|0\rangle_E + \sqrt{p}\,|1\rangle_R|2\rangle_E$$

Rock quickly becomes entangled with photon path & decoheres.

Note: rock is <u>heavy</u>, $\therefore$ not affected much when a photon bumps into it.

That's why we can model the rock states $|0\rangle$ and $|1\rangle$ as unaffected (other than entanglement). An approximation, obviously.

Decoherence helps explain the absence of macroscopic quantum superpositions in the world.

E.g. Schrödinger's cat. The cat states $|\text{awake}\rangle / |\text{asleep}\rangle$ interact differently with photons & air in the box. If we ignore those environmental degrees of freedom (trace over them), the cat state is a mixed density matrix, not a pure state. (We'll talk later about whether "ignoring" the environment is sensible.)

Same goes for people, planets, etc. — quickly decohere in spatially-localized states.

Saturn's moon Hyperion – tumbles chaotically end-over-end. Would look like a quantum blob if it weren't for decoherence via environmental monitoring.

Classicality

You may have been told that the classical limit of QM is "action much larger than $\hbar$," or something about Ehrenfest's Theorem. Those are relevant, but decoherence does the real work.

E.g. Ehrenfest: consider a nonrelativistic particle with position x and potential $V(x)$. Then

$$m \frac{d^2}{dt^2} \langle x \rangle = -\left\langle \frac{dV}{dx} \right\rangle ,$$

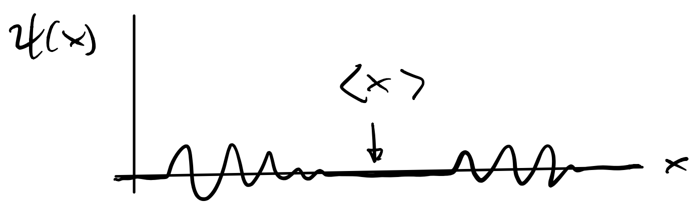
where $\langle \cdot \rangle = \langle \psi | \cdot | \psi \rangle$. Always true - it's a theorem. But if we have a "small wave packet," this becomes

$$m \frac{d^2}{dt^2} \langle x \rangle = - \frac{dV(\langle x \rangle)}{dx} .$$

Now it looks like Newton's 2^{nd} Law, $F = ma$!

Great, but why should we have a small wave packet? Wave packets generally spread over time.

Or: what if particle is delocalized?



In this case Ehrenfest's Theorem is still true, but $\langle x \rangle$ doesn't say much about where the particle is likely to be found.

Need decoherence to explain why objects seem to have definite positions.

Pointer Bases

We remarked that our dephasing unitary picked out the $\{|0\rangle, |1\rangle\}$ basis as special (by construction). Consider expressing the same qubit state in the $|\pm\rangle$ basis:

$$\begin{pmatrix} |+\rangle \\ |-\rangle \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}_{\hat{R}} \begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$$

The density matrix $\hat{\rho} = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$ becomes

$$\hat{\rho}^{(\pm)} = \hat{R}\hat{\rho}\hat{R}^{+} = \frac{1}{2} \begin{pmatrix} \rho_{00} + \rho_{01} + \rho_{10} + \rho_{11} & \rho_{00} - \rho_{01} + \rho_{10} - \rho_{11} \\ \rho_{00} + \rho_{01} - \rho_{10} - \rho_{11} & \rho_{00} - \rho_{01} - \rho_{10} + \rho_{11} \end{pmatrix}$$

Use $\rho_{11} = 1 - \rho_{00}$, and set $\rho_{10}, \rho_{01} \to 0$:

$$\hat{\rho}^{(\pm)} \to \begin{pmatrix} 1 & 2\rho_{00} - 1 \\ 2\rho_{00} - 1 & 1 \end{pmatrix}.$$

Nothing particularly diagonal about that.

(But it is nevertheless decohered.)

There's nothing special about a diagonal density matrix. Every matrix is diagonal in some basis.

Indeed, given any system + environment in a pure state $|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_E$, there is always a Schmidt basis:

$$|\Psi\rangle = \sum_n \psi_n |\sigma_n\rangle_S \otimes |\lambda_n\rangle_E$$

The reduced density matrix $\hat{\rho}_S = \mathrm{Tr}_E |\Psi\rangle\langle\Psi|$ is automatically diagonal in this basis.

What's special is that, given any particular quantum channel that can cause decoherence, there is a preferred basis such that $\hat{\rho}_s(t)$ evolves toward becoming diagonal in that basis.

This preferred basis is the pointer basis.

For our dephasing qubit, the pointer states are obviously $\{|0\rangle, |1\rangle\}$.

"Pointer basis" and "Schmidt basis" are not synonyms!

Schmidt basis exists for any pure state in a bipartite system, and reduced density matrices for either factor are automatically diagonal therein.

whereas pointer states are the fixed states in which $\hat{\rho}_s$ becomes increasingly diagonal as the system & environment interact.

Often $\hat{\rho}_s$ approaches diagonality in the pointer basis, but doesn't exactly get there – decoherence is approximate. (But extremely good.)

So the idea is that the time-dependent Schmidt basis, in which $\hat{\rho}_A(t)$ is exactly diagonal, evolves toward the fixed pointer basis.

Of course such behavior is <u>highly non-generic</u>.

Requires something special about the initial setup — in particular, no initial entanglement between $\mathcal{H}_A$ & $\mathcal{H}_E$, and a very large environment: $\dim \mathcal{H}_E \gg \dim \mathcal{H}_A$.

Indeed, for <u>any</u> $|\Psi(T)\rangle$ (and therefore any $\hat{\rho}_A(T) = \mathrm{Tr}_E |\Psi(T)\rangle\langle\Psi(T)|$), there is a state $|\Psi(0)\rangle = \hat{\mathcal{U}}^{-1}(T)|\Psi(T)\rangle$ from which it could have unitarily evolved.

The ubiquity/naturalness of decoherence relies on the special (low-entropy, far-from-equilibrium) state of our world.

If a system <u>starts</u> in a pure state that is a pointer state for some quantum channel, it <u>stays pure</u> – no entanglement is created.

E.g for dephasing, we can write the evolution as

$$|0\rangle_A|0\rangle_E \to |0\rangle_A|\eta_0\rangle_E, \quad |1\rangle_A|0\rangle_E \to |1\rangle_A|\eta_1\rangle_E,$$

where

$$|\eta_0\rangle_E = \sqrt{1-p}\,|0\rangle_E + \sqrt{p}\,|1\rangle_E$$
$$|\eta_1\rangle_E = \sqrt{1-p}\,|0\rangle_E + \sqrt{p}\,|2\rangle_E$$

Note $\{|\eta_0\rangle, |\eta_1\rangle\}$ aren't orthogonal: $\langle\eta_0|\eta_1\rangle = 1-p$.

If we start with our qubit in $|0\rangle$,

$$\hat{\rho}(0) = |0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_E,$$

we evolve in one timestep to

$$\hat{\rho}(T) = |0\rangle\langle 0|_A \otimes |\eta_0\rangle\langle\eta_0|_E,$$

and the reduced density matrix for A is still $\hat{\rho}_A(T) = |0\rangle\langle 0|_A$. No decoherence.

Physically, pointer states are those that are "robust" under monitoring by the environment - they don't rapidly become entangled.

E.g. spatially localized states of a dust particle or Schrödinger's cat.

Think of a general system + environment, $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$, with a Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_E + \hat{H}_I \; .$$

S & E may evolve by themselves under $\hat{H}_S$ and $\hat{H}_E$, but entanglement can only be generated by $\hat{H}_I$.

A simple criterion for states $\{|\phi_a\rangle_S\}$ to be pointer states is that they are <u>eigenstates of $\hat{H}_I$</u>. Defining $\hat{\pi}_a = |\phi_a\rangle\langle\phi_a|$, that's equivalent to

$$[\hat{H}_I, \hat{\pi}_a] = 0 \, .$$

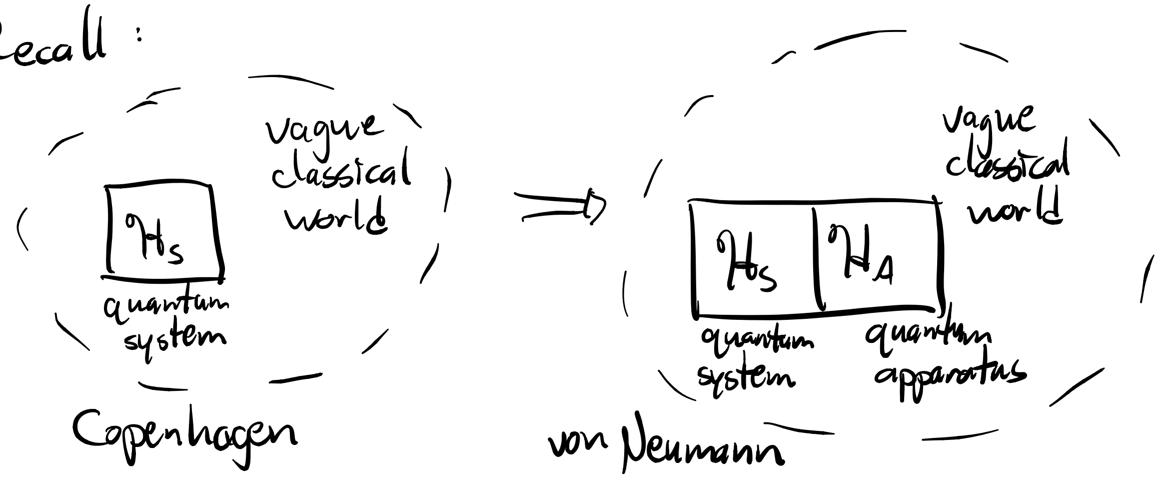
Zurek's "commutativity criterion."

Decoherence & Measurement

Does decoherence solve the measurement problem? (I.e., explain what a quantum measurement truly is, and why it seems different from unitary Schrödinger evolution?)

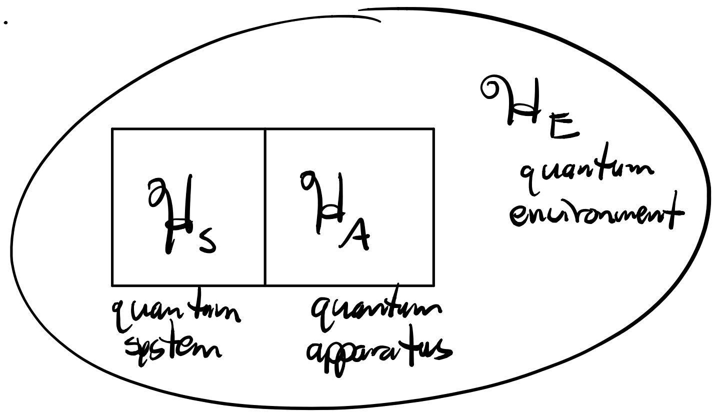
No. But it is very relevant for the discussion.

Recall:



Von Neumann explained measurement through entanglement between system and apparatus, but "observe the apparatus" was still magic.

Decoherence suggests that we can eliminate the "vague classical world" once and for all.



The new progress comes from taking seriously the rest of the world; there is only one wave function.

We now have

$$|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E .$$

Decoherence explains why the reduced density operator $\hat{\rho}_{SA} = \mathrm{Tr}_E |\Psi\rangle\langle\Psi|$ evolves from pure to mixed, where in each component there is a definite read-out on our macroscopic apparatus. (That's why they're called "pointer states"!)

Bohr: $\sum_n \psi_n |\phi_n\rangle_S \rightsquigarrow |\phi_n\rangle$ w/ prob. $|\psi_n|^2$

von Neumann: $\left(\sum_n \psi_n |\phi_n\rangle_S\right)|\eta_0\rangle_A \rightsquigarrow \sum_n \psi_n |\phi_n\rangle_S |\eta_n\rangle_A$

$\rightsquigarrow |\phi_n\rangle_S |\eta_n\rangle_A$ w/ prob $|\psi_n|^2$

Decoherence: $\left(\sum_n \psi_n |\phi_n\rangle_S\right)|\eta_0\rangle_A |e_0\rangle_E$

$\rightsquigarrow \left(\sum_n \psi_n |\phi_n\rangle_S |\eta_n\rangle_A\right)|e_0\rangle_E$ "measurement"

$\rightsquigarrow \sum_n \psi_n |\phi_n\rangle_S |\eta_n\rangle_A |e_n\rangle_E$ "decoherence"

$\rightsquigarrow |\phi_n\rangle_S |\eta_n\rangle_A |e_n\rangle_E$ w/prob. $|\psi_n|^2$.

But — that doesn't "solve the measurement problem."

The reduced density op. $\hat{\rho}_{SA}$ is mixed, and therefore <u>looks</u> just like a classical statistical ensemble of possibilities.

But so what? It's <u>not</u> such an ensemble — it's just a factor within a larger entangled state.